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The Study of Nonlinear Dynamical Systems Nuclear Fission Using Hurwitz Criterion

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Abstract In this paper, the nonlinear dynamic system of equations, a type of nuclear fission reactor is solved analytically and numerically. Considering that the direct solution of three-dimensional dynamical systems analysis and more in order to determine the stability and instability, in terms of algebraic systems is difficult. Using certain situations in mathematics called Hurwitz criterion, Necessary and sufficient conditions for a stable dynamical system is determined and the parameters that most influence the quality of the dynamic behavior of a nuclear fission reactor have been determined.

Keywords nonlinear dynamical system · Hurwitz criterion · stability

Mathematics Subject Classification (2010) 37N20

1 Introduction

Analysis of nonlinear dynamic systems, such as nuclear fission reactors to determine quantitative parameters on stability and instability of the system is important. In order to evaluate dynamical system the number of variable parameters of time-depended increase that in this case analytical solution of the algebraic is problem. In study, in such cases by imposing certain conditions on the math simple, we called the Hurwitz criterion the system are analyzed. In this paper we consider time-dependent parameters of a nuclear fission reactor system, an analytical method are used and the numerical results are compared with the analytical results were confirmed.

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2 Materials and methods

In order to describe the dynamics of a system for the study of nuclear fission, it must be considered with time-varying parameters. These parameters can be summarized as

$$X_i = N_n, N_d, T_f, T_c, N_f, ... \quad (1)$$

Where N_n the neutron density, N_d delayed neutron precursor density, T_f fuel temperature,T_c coolant temperature and N_f fuel density. Since fuel constantly fed to nuclear fission reactor systems, so be sure that the system is stable, and self-help should be investigated. The complete nonlinear fission dynamic given in the form

$$\frac{dX_i}{dt}|_{X=X^*} = f_i, \qquad Number of state variables; i = 1, 2, ..., k \quad (2)$$

Usually these equations are coupled with nonlinear equations.

2.1 The analytical method

Analysis of the equations (2) and Stability of system must be obtaining fixed points system dynamic equations in state space with the following conditions:

$$\frac{dX_i}{dt}|_{X=X^*} = 0 \quad ; i = 1, 2, ..., k \quad (3)$$

In order to investigate the stability of a nuclear system a small perturbation insert to system, whether system will return to his consistent point or not. For a δX small perturbation in a way that $X = X^* + \delta X$ here δX the following condition holds.

$$\frac{d(\delta X)}{dt} \cong J(\delta X) \quad (4)$$

And J corresponding Jacobean, given by the following equation

$$J_{i,j} = [\frac{df_i}{dX_j}]_{X=X^*} \quad (5)$$

f_i is function i on the right side of the dynamic equation. Equation (5) for the small changes that δX has enough precise.

$$\delta X_i = a_{i,1}exp(\lambda_1 t) + a_{i,2}exp(\lambda_2 t) + a_{i,3}exp(\lambda_3 t) + ... \quad (6)$$

$a_{i,j}$ are constants that depend on Jacobean and initial conditions on x^* direction are and λ_j for values of $j = 1, 2, ..., n$, the eigenvalues. Determine the eigenvalues of the Jacobean characteristic equation (5) is obtained:

$$det(J - \lambda I) = 0 \quad (7)$$

Characteristic equation is an equation of degree n in terms of λ . Solutions of this equation can be $\lambda_j = Re_j + iIm_j$. Therefore, the sign of the real

part eigenvalues define a particular dynamic property. If λ_j are all negative, according to equation (6) after time t ,δX goes to zero, and the system is stable, but if one of λ_j are positive, δX after time t will be great and the system becomes unstable. According to the Hurwitz criterion, a necessary and sufficient condition for the stability of a dynamical system is determined, so that all eigenvalues have a negative real part.
For an nth-degree polynomial

$$\alpha_0 z^n + \alpha_1 z^{n-1} + ... + \alpha_{n-1} z + \alpha_n = 0 \tag{8}$$

In this case, if α_0 the coefficient is positive and the other coefficients are real, the Hurwitz criterion stated that necessary and sufficient condition for have negative real roots of the equation (8), is written as

$$\Delta = \begin{vmatrix} \alpha_1 & \alpha_2 & 0 & 0 & \cdots 0 \\ \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 & \cdots 0 \\ \alpha_5 & \alpha_4 & \alpha_3 & \alpha_2 & \cdots 0 \\ & \vdots & & & \\ \alpha_{2n-1} & \alpha_{2n-2} & \cdots & \cdots & \cdots \alpha_n \end{vmatrix} > 0 \tag{9}$$

2.2 Numerical Methods

Another way to evaluate the stability and instability of dynamical system applied, using numerical method. Thus, by knowing all the physical parameters of the reactor (or any nuclear system) with the identified power,the system dynamic equations using a conventional numerical methods such as Runge-Kutta [8] or by using mathematical software MATLAB, the equations are solved and the stability or instability of the system at different times for perturbation entering to the system for the dynamic variables change over time and to change them to each analyzed .

3 Describing the dynamics of a fission reactor systems

In order to describe the dynamics of a system variable fission reactor four basic modes of behavior of the reactor is considered.

$$\frac{dN_n(t)}{dt} = kN_N(t) + a_1 N_n(t) T_f(t) + a_2 N_n(t) T_c(t) + \lambda_d N_d(t) \tag{10}$$

$$\frac{dN_d(t)}{dt} = a_3 N_n(t) - \lambda_d N_d \tag{11}$$

$$\frac{dT_f(t)}{dt} = \gamma N_n(t) - \frac{T_f(t) - T_c(t)}{\tau_f} \tag{12}$$

$$\frac{dT_c(t)}{dt} = \frac{T_f(t) - T_c(t)}{\tau_f} - \frac{T_c(t) - T_{\sin k}}{\tau_c} \tag{13}$$

Here N_n the neutron density, N_d delayed neutron precursor density; T_f fuel temperature and T_c coolant temperature are parameters of fission system. In equations 10, a_1 is always negative, while a_2 is depending on the type and structure of the coolant of reactor and can be positive or negative, τ_f is the mean residence time of thermal energy in the fuel. T_{sink} is the reference temperature of the heat exchanger and τ_c is the mean residence time of thermal energy in the coolant.

4 Analytical study of dynamical systems using the Hurwitz criterion

For fourth-order polynomials

$$p_0\lambda^4 + p_1\lambda^3 + p_2\lambda^2 + p_3\lambda + p_4 = 0 \tag{14}$$

The Hurwitz criterion for equation (10) can be expressed as:

$$\frac{p_1}{p_0} > 0;\ \frac{p_2}{p_0} > 0;\ \frac{p_3}{p_0} > 0;\ \frac{p_4}{p_0} > 0;\ ,\ p_3(p_1p_2 - p_0p_3) - (p_1^2p_4) > 0 \tag{15}$$

Eqs. 3, 5 and 7 can be combined to obtain Characteristic equation, A detailed examination of the eigenvalue reveals that the zero-power fixed point possesses two negative pure real eigenvalues and one positive pure real eigenvalue. This indicates that the zero- power fixed point is unstable. Our analysis of the eigenvalue of the full-power fixed point suggests that the energy mean residence time in the fuel τ_f and the coolant τ_c, affects the eigenvalues in a most significant manner. That is, the dynamic represented by equations (10)-(13) are particularly sensitive to changes in τ_f and τ_c. The coefficients of characteristic equations for the fixed point corresponding to the full-power fixed point for $a_2 < 0$ mode include

$$p_0 = 1;\ \ p_2 = \varOmega(600.4\tau_c^2{+}75.08\tau_f^2{+}530.48\tau_f\tau_c{+}4\tau_c{+}\tau_f);\ \ p_4 = \varOmega(6.4\tau_f{+}25.6\tau_c)$$

$$p_1 = \varOmega(8\tau_c^2{+}75.08\tau_f^2\tau_c{+}300.32\tau_c^2\tau_f{+}6\tau_f\tau_c);\quad p_3 = \varOmega(6.4\tau_f\tau_c{+}620.32\tau_c{+}155.08\tau_f)$$

And coefficients for $a_2 > 0$ mode include

$$p_0 = 1;\qquad p_1 = \mu(4\tau_c^2 - 75.08\tau_f^2\tau_c + 150.16\tau_c^2\tau_f - \tau_f^2);\qquad p_4 = \frac{1}{\tau_f\tau_c}$$

$$p_2 = \mu(300.32\tau_c^2{-}75.08\tau_f^2{-}50\tau_c\tau_f{+}2\tau_c{-}\tau_f);\ \ p_3 = \mu({-}4\tau_f\tau_c{+}250.16\tau_c{-}125.08\tau_f)$$

Here $\varOmega = \tau_f\tau_c(4\tau_c + \tau_f)$ and $\mu = \tau_f\tau_c(2\tau_c - \tau_f)$ are given in terms of previously defined system parameters.

Applying the criteria of Horowitz (relations (15)) for the characteristic equation, we conclude that the full potential of the fixed point $a_2 < 0$ is always a negative real roots for all values of the system parameters, which is stable for the case of $a_2 > 0$ the condition of stability stands $\frac{\tau_f}{\tau_c}$ for the dynamic case.

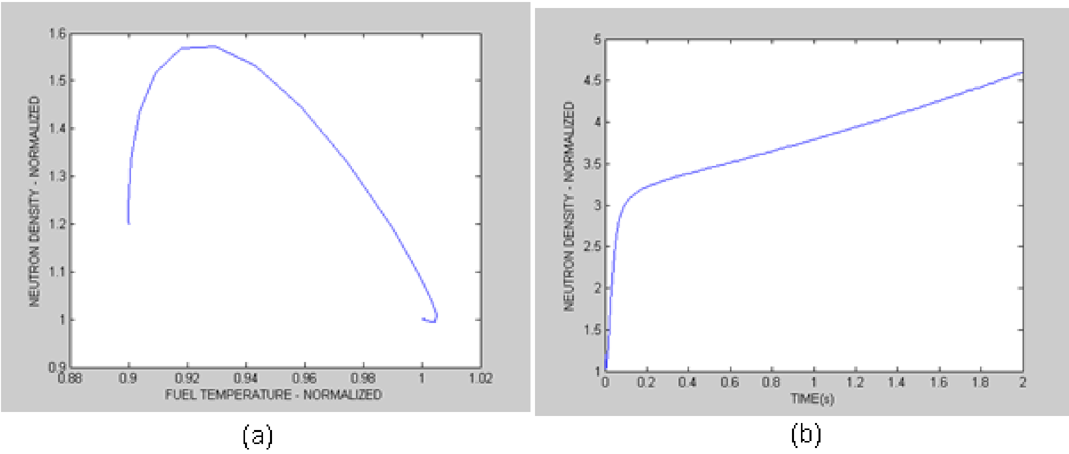

Fig. 1 Variations of neutron density and temperature of the fuel for(a) $a_2 < 0$, $\frac{\tau_f}{\tau_c} = 15$, $\tau_c = 0.01$ and (b) $a_2 < 0$, $\frac{\tau_f}{\tau_c} = 1.8$, $\tau_c = 0.02$

Table 1 Parameter values used in the calculations for a nuclear fission reactor type [8].

τ_f-Variable	$k = \pm 85 s^{-1}$	$a_1 = 0.1^0 cs^{-1}$	$a_2 = 0.3^0 cs^{-1}$	$a_3 = 75 s^{-1}$
τ_c-Variable	$\lambda_d = 0.08 s^{-1}$	$\gamma = 3.4 \times 10^{-6} cm^3 s^{-1} {}^0 c$	$T_{sink} = 200^0 c$	

4.1 Numerical study of dynamical systems

In numerical method, based on non-linear dynamic equations for a nuclear fission reactor, with typical constant values for system in Table 1 the equations are solved by using MATLAB mathematical of software and system stability and in stability conditions for perturbation are introduced at different times have been considered. The numerical results in Figure 1 are presented. Results indicate stable and unstable dynamical system forth numerical solution with the analytical results is the same.

5 Conclusion

In order to describe nuclear fission reactor systems, nonlinear dynamic formulation with respect to almost all parameters affecting the behavior of fission reactors, is presented. According to those equations, stable and unstable dynamical system using the criteria of Horowitz as analytical and using the mathematical software MATLAB as numerical solutions is presented. Result show that the energy means residence time in the fuel τ_f and the coolant τ_c are parameter for the evolution of our fission system.

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