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Performance Evaluation in the Presence of Heterogeneous Indicators in Data Envelopment Analysis: A Case Study on Top Investment Companies of Tehran Stock Exchange

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Abstract Evaluating the performance of organizations can provide managers with useful information about the status of the organization compared to other organizations so that managers can take a step towards the growth and excellence of the organization. Obviously, the number of indicators and their amount affect the performance evaluation of organizations. So, by collecting the exact values of the indicators, an accurate and accurate performance evaluation will be provided to managers of organizations. In this article, we intend to evaluate the companies investing in the stock exchange. Since in the table of indices related to these companies published by the Iran Stock Exchange Organization there are indices whose values have been lost for any reason (not available - heterogeneous index), it is necessary to use envelopment analysis models. We used data (DEA) in the presence of heterogeneous indicators. We have used the model of Cook et al. [3] article to evaluate companies. For the conceptual use of research, we have described and implemented their method step by step. Lastly, we have analyzed the results.

Keywords Non-homogeneous indicators · Investment Companies · Tehran Stock Exchange · Performance Evaluation · Data Envelopment Analysis

Mathematics Subject Classification (2010) 65L03 · 60J74 · 60J76

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1 Introduction

Data Envelopment Analysis (DEA) has been proposed by Charnes et al. [4], which is a method for evaluating the relative efficiency of homogeneous unit decision sets (DMUs). This means that each has the same number of inputs and outputs. Mohamadi et al. [10] have dealt with the effect of measuring the efficiency of manufacturing and industrial organizations and comparing the efficiency between their units, which is one of the most important issues in the field of industry today. They used a combination of gray relationship analysis and data envelopment analysis techniques to evaluate suppliers at Faragostar Toos. Thompson et al. [13] evaluated performance in the absence of uniformity (heterogeneity or heterogeneity) of DMU values. In their article, they address the problem of lost data; That a DMU produces a specific output, but its value cannot be known. One approach to solving this problem that is used in their paper is to ”create” a value for the lost output; for example, the average of known values can be used to fill in the gaps. Also, the value of zero replaces the lost value. Cook et al. [2] examined a simple case in which DMUs appeared in a 2-group cluster. They provided a clear and simple solution for the two groups. Cook et al. [3] developed the DEA model for missing outputs, in which DMUs consisted of unique subgroups. Li et al. [9] evaluated heterogeneity in the input status in measuring the environmental importance of Chinese provinces, given that not all provincial inputs are of the same nature (heterogeneous indicators). Undesirable outputs may also be heterogeneous. To solve this problem, Podinovski et al. [12] proposed a non-parametric method. Du et al. [5] considered heterogeneous DMUs in parallel network structure. Ji et al. [8] examined the heterogeneous DEA method for assessing China’s sustainable urbanization. Wu et al. [7] proposed a heterogeneous DEA model, which examines the problem of homogeneous asynchronous inputs. DMU outputs are intended to measure the environmental importance of China’s industrial sectors. The status of outputs and inputs is such that a company may have a lost output and inputs. They developed the DEA model for heterogeneous inputs and outputs. Inspired by the ideas of heterogeneous articles, we intend to evaluate investment companies. In the continuation of this research, we will describe the tools and methods used and at the end, we will discuss some research findings.

2 Tools and Methods

In this section, we intend to summarize the model of Cook et al. [3] in an algorithm. The proposed algorithm, in Section 3, will be fully described with a practical example. Heterogeneous DEA algorithm in output with homogeneous inputs:

Step (1): Divide the companies into N_p $(p = 1, \ldots, P)$ categories or groups based on the output generated.

Step (2): Specify R_k $(k = 1, \ldots, K)$, which represents the subset of the

outputs so that its members appear as the output of exactly one N_p class of DMUs.
Step (3): Specify L_{N_p}, which represents R_k, which specifies the set of all outputs for each DMU in N_p.
Step (4): Decide to allocate part of the i-th input to each of the output subgroups $L_{N_{pj}}$ for DMU_j with $\alpha_{iR_{kpj}}$ are displayed.
Step (5): Define the performance of DMUs in each subgroup R_k.
Step (6): Weight average of the performance score of the subgroup to obtain the overall performance of the units. To obtain the overall efficiency score of the units, use the following model, proposed by Cook et al. [3].

$$
\begin{aligned}
e_o = max \ & \sum_{R_k \in L_{N_{po}}} \sum_{r \in R_k} \mu_r y_{rj_o} \\
s.t. \ & \sum_{R_k \in L_{N_{po}}} (\sum_{i \in R_k} \gamma_{iR_k p^o} x_{ij_o}) = 1 \\
& \sum_{r \in R_k} \mu_r y_{rj} - \sum_{i \in R_k} \gamma_{iR_k p} \ \ x_{ij} \leq 0, \forall j \in N_p, R_k \in L_{N_p}, p = 1, \ldots, P \\
& \sum_{R_k \in L_{N_p}} \gamma_{iR_k p} = v_i, \forall i, p = 1, \ldots, P \\
& v_i a_{iR_k p} \leq \gamma_{iR_k p} \leq v_i b_{iR_k p}, \forall i, R_k \in L_{N_p}, p = 1, \ldots, P \\
& \mu_r, v_i, \gamma_{iR_k p} \geq \epsilon, \forall i, r, R_k \in L_{N_p}, p = 1, \ldots, P
\end{aligned}
\tag{1}
$$

Where x_{ij} and y_{rj} represent the input and output of the units under evaluation, μ_r is the weight of the output, and $\gamma_{iR_k p}$ depend on the percentage of consumption of inputs and weights which is completely described in Cook et al. (2013).

Definition 1 In the evaluation of DMU_o; $o \in \{1, \ldots, n\}$, by solving model (1), if in optimality $e_o = 1$ we say DMU_o is efficient, otherwise we call DMU_o inefficient.

3 Findings

In the following, we intend to find the proposed algorithm to find efficient units in the presence of heterogeneous indices with a practical example.
Study companies:
The statistical population of the study includes 32 active investment companies in Tehran Stock Exchange, whose names are given in Table (1).

Variables:
Variables include independent variables (input and output indicators) and dependent variables (performance score).
Input indicators:
Debt Ratio - Receivables Collection Period
Output indicators:
Operating profit margin - Profit to special interest - Return on assets - Return on capital - Current ratio - Instant ratio (Fast ratio) - Liquidity ratio.
According to MciNel et al. [11] Horcher et al. [6], Walter et al. [14] and Brown et al. [1], the above indicators can be defined as follows.

Table 1 Names of the studied companies

1	Iranian Industries Investment	23	Pardis Investment	12	Arzesh Afarinan Pasargad
2	Insurance Industry Investment	24	Planners of Fars and Khuzestan Investment	13	Arian Economic Development
3	Investing in mining Investment	25	Northern Development Investment	14	Takado
4	Iranian Plateau Investment	26	Industrial Development of Iran Investment	15	Atieh Damavand Investment
5	Behshahr Group Investment	27	Goharan Omid Investment	16	Value Creators Investment
6	Pars industry standard Investment	28	National Development Investment	17	Etela Alborz Investment
7	National Investment	29	Jami Investment	18	Novin Economy Investment
8	Noor Kowsar Iranian Investment	30	Kharazmi Investment	19	Maskan Bank Investment
9	Electrical Industry Staff Investment Company Zanjan and Qazvin	31	Danayan Pars Investment	20	Bahman Investment
10	Expansion of investment by Iranians	32	Saipa Investment	21	Buali Investment
11	Sepah Investment	22	Pars Arian Investment		

Table 2 Concepts of input and output indicators

Definition	Indicator title
Total debts of the company	Current debt +long-term debt
Property	Current assets + fixed assets + other assets
Debt ratio	$\frac{\text{Total debt companies}}{\text{Assest}}$
Periodicals Collection	The average number of days required to convert business receivables into cash
profit margin	$\frac{\text{operating profit}}{\text{Sales}}$
Specific profit (gross profit)	Cost of goods sold − the amount of goods sold
Special interest (net profit)	Various administrative expenses, sales and advertising as well as taxes and other financial expenses) - Special profit (Gross profit)
Return on assets	$\frac{\text{Annual profit}}{\text{Total assets of the company}}$
Return on investment	$\frac{\text{Annual profit}}{\text{Average total assest}}$
Current ratio	$\frac{\text{Current assest}}{\text{Current debt}}$
Instant ratio	$\frac{\text{Current assest - Inventory}}{\text{Current debt}}$
Liquidity ratio	$\frac{\text{Cash}}{\text{Current debt}}$

Indicator values for 32 companies are given in Table (3). The data was extracted from the site ”codal.ir”.

In this section, we will implement the algorithm presented in the previous section step by step.

Step (1): According to the values of the indicators in Table (3) and the missing indicators, $P = \{1, 2, 3, 4, 5\}$ and we have:

$N_1 = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7\}, \qquad N_2 = \{y_1, y_3, y_4, y_5, y_6\},$

Table 3 Values of input and output indicators

DMU	x1	x2	y1	y2	y3	y4	y5	y6	y7
1	0.3	513.95	98.88	84.54	9.9	17.13	2.64	2.64	2.1
2	0.19	0.1	51.26	——-	3.81	4.87	0.03	0.03	——
3	0.75	2,763.06	72.78	62.17	2.99	33.6	1.05	0.79	0.09
4	0.5 9	80.62	97.07	102.12	16.28	81.63	3.62	3.62	0.14
5	0.02	92.72	97.02	97.22	38.81	76.08	39.63	39.63	35
6	0.12	226.19	106.81	106.89	18.78	——-	8.45	8.45	7.5
7	0.63	193.49	93.48	90.37	11.86	51.23	1.53	1.53	1.42
8	0.63	240.66	98.39	103.47	17.71	———	4.44	4.15	2.97
9	0.17	279.82	111	117.58	25.61	49.06	2.5	2.5	1.5
10	0.07	52.79	90.05	90.19	24.75	40.68	10.4	10.4	9.57
11	0.26	506.87	99.21	79.59	13.5	23.88	1.82	1.74	0.81
12	0.07	115.43	97.63	97.63	37.97	72.44	14.73	14.73	12.82
13	0.19	153.32	81.93	75.38	14.95	26.8	2.94	2.94	2.36
14	0.26	461.14	93.34	72.26	14.6	24.57	3.71	3.57	2.56
15	0.1	95.8	118.21	124.32	27.17	62.05	5.44	5.44	4.82
16	0.92	418.68	100	81.74	7.99	——-	0.65	0.65	0.24
17	0.12	85.82	98.91	99.35	28.16	68.48	6.72	6.72	6.18
18	0.14	25.3	97.08	97.08	35.9	80.34	5.28	5.28	5.09
19	0.29	327.39	100.92	89.85	14.33	41.45	1.62	1.09	0.38
20	0.19	223.08	97.09	76.79	11.79	22.36	4.79	4.79	4.29
21	0.41	1,058.57	107.27	92.72	21.32	57.33	1.99	1.99	0.23
22	0.13	120.42	99.63	98.58	29.03	53.22	6.45	6.45	5.53
23	0.44	0.1	——-	———	21.02	——	2.1	2.09	1.37
24	0.35	349.92	110.65	77.29	6.09	11.73	0.68	0.68	0.46
25	0.05	99.34	175.66	174.65	41.92	115.25	17.3	17.3	15.5
26	0.65	256.13	96.78	100.76	12.31	53.64	0.6	0.58	0.06
27	0.07	222.71	96.89	88.59	41.56	89.99	13.27	13.27	8.88
28	0.04	0.1	———	———	0.03	0.03	1.48	1.48	0.49
29	0.15	207.12	98.01	96.76	20.6	60.56	0.98	0.98	0.07
30	0.16	112.31	98.28	92.98	25.96	46.86	8.06	8.06	7.37
31	0.04	361.41	82.68	79.38	21.98	34.61	13.47	13.46	4.66
32	0.53	489	86.8	44.76	12.27	32.46	0.89	0.89	0.2

$N_3 = \{y_1, y_2, y_3, y_5, y_6, y_7\}, \quad N_4 = \{y_3, y_5, y_6, y_7\}, \quad N_5 = \{y_3, y_4, y_5, y_6, y_7\}.$
Step (2): According to the categories of N_p specified above, $R_k \ (k = 1, \dots, 5)$ is as follows: $R_1 = y_1, \ \ R_2 = y_2, \ \ R_3 = y_3, y_5, y_6, \ \ R_4 = y_4, \ \ R_5 = y_7$.
Step (3): According to the categories N_p, $(p = 1, 2, 3, 4, 5)$ and $R_k \ (k = 1, \dots, 5)$ specified above$L_(N_p)$ is determined to be as follows:
$L_{N_1} = \{R_1, R_2, R_3, R_4, R_5\}, \ \ L_{N_2} = \{R_1, R_3, R_4\},$
$L_{N_3} = \{R_1, R_2, R_3, R_5\}, \ \ L_{N_4} = \{R_3, R_5\}, \ \ L_{N_5} = \{R_3, R_4, R_5\}.$
Step (5): The weights of assigning inputs are determined by solving the model, but if the expert intends to use the percentages he wants to produce each batch of output, he can assign the weights of assigning each input to produce each batch of output. Introduce and enter the model.
Step (6): Define the efficiency of DMUs in each subgroup R_k.
For example, the efficiency definition of DMU_2 would be as follows. Since DMU_2 is the category booklet N_2 and $L_{N_2} = \{R_1, R_3, R_4\}$ so we have:

$$e^2_{R_1} = \frac{\sum_{r\in R_1} u_r y_{r_2}}{\sum_i v_i \alpha_{iR_1 2} x^2_i}, \quad e^2_{R_3} = \frac{\sum_{r\in R_3} u_r y_{r2}}{\sum_i v_i \alpha_{iR_3 2} x_{i2}}, \quad e^2_{R_4} = \frac{\sum_{r\in R_4} u_r y_{r2}}{\sum_i v_i \alpha_{iR_4 2} x_{i2}},$$

where $\alpha_{iR_{k2}}, \ k = 1, 3, 4$ assigned weight of inputs to produce outputs of each category $R_k, \ k = 1, 3, 4$ is under evaluation DMU_2 unit.
For example, for DMU_2 the overall efficiency is as follows:

$$e_2 = w_1 e^2_{R_1} + w_2 e^2_{R_3} + w_3 e^2_{R_4} = w_1 \frac{\sum_{r\in R_1} u_r y_{r2}}{\sum_i v_i \alpha_{iR_1 2} x_{i2}} + w_2 \frac{\sum_{r\in R_3} u_r y_{r2}}{\sum_i v_i \alpha_{iR_3 2} x_{i2}} + w_3 \frac{\sum_{r\in R_4} u_r y_{r2}}{\sum_i v_i \alpha_{iR_4 2} x_{i2}}$$

$$= w_1 \frac{u_1 y_{12}}{v_1 \alpha_{1R_1 2} x_{12} + v_2 \alpha_{2R_1 2} x_{22}} + w_2 \frac{u_3 y_{32} + u_5 y_{52} + u_6 y_{62}}{v_1 \alpha_{1R_3 2} x_{12} + v_2 \alpha_{2R_3 2} x_{22}} + w_3 \frac{u_4 y_{42}}{v_1 \alpha_{1R_4 2} x_{12} + v_2 \alpha_{2R_4 2} x_{22}},$$

Table 4 Results of implementation model (1)

DMU_j	1	2	3	4	5	6	7	8
e_j^*	0.7183	0.1665	0.903	1	0.0716	0.0222	0.7686	0.1068
DMU_j	9	10	11	12	13	14	15	16
e_j^*	0.3533	0.6549	0.1128	0.258	0.6685	0.1104	0.8143	0.2333
DMU_j	17	18	19	20	21	22	23	24
e_j^*	0.1602	0.0532	0.4149	0.0686	0.2276	0.16	1	0.544
DMU_j	25	26	27	28	29	30	31	32
e_j^*	0.0884	0.415	0.4075	0.2545	0.1448	0.4312	0.1513	1

Table 5 Results obtained from the model CCR

DMU_j	1	2	3	4	5	6	7	8
$\theta - j$	0.03949	0.41673	0.00252	0.03503	1	0.17377	0.06188	0.08454
DMU_j	9	10	11	12	13	14	15	16
θ_j	0.18592	0.74722	0.05613	0.66848	0.15956	0.06541	0.49379	0.02412
DMU_j	17	18	19	20	21	22	23	24
θ_j	0.51187	1	0.08037	0.0982	0.04467	0.41487	1	0.03074
DMU_j	25	26	27	28	29	30	31	32
θ_j	0.89275	0.05619	0.42691	1	0.19341	0.35781	0.28317	0.04312

Where $w_1 + w_2 + w_3 = 1$. By implementing model (1), the efficiency of each unit is obtained, the results of which are given in Table (4). According to the obtained efficiency values the units DMU_4, DMU_{23}, DMU_{32} are efficient with a value equal to one, and the other units are inefficient.

According to the results obtained in Table (4), the units that have an efficiency score equal to one are efficient units in the presence of heterogeneous indicators. Units 4, 23 and 32 are efficient units and the rest are ineffective. If a base DEA model is used instead of the heterogeneous model, so that all units are evaluated, it can be said that the indicators y_2, y_4, y_6 and y_7, which include missing indicators, are removed from the outputs. In this case, we will reach the results of Table (5).

According to the results obtained above, the units were 5, 18, 23 and 28 eeffectively identified and according to the results obtained in Table 4, only one unit has been correctly identified as effective. Therefore, it is not recommended to remove missing indicators in the evaluation of units.

4 Discussion and Conclusion

Since the basic DEA models are effective for non-negative homogeneous data, if we want to evaluate the units, we can consider the following using data envelopment analysis models. In the first case, the unit with the missing index must be excluded from the calculation, in this case the unit is not evaluated. In the second case, an index such as a missing index should be removed from all units. In this case, the correct assessment has not been done. In the third case, one can use the models related to heterogeneous indicators, which are discussed in this article. This article evaluates the top 32 investment companies of Tehran Stock Exchange. Since the values of some indicators have been lost for any reason (i.e. their numerical value has not been published by the Exchange Organization), we decided to use the data envelopment analysis model with non-homogeneous indicator to evaluate companies. First, an algorithm was proposed for the article model of Cook et al. [3] and then a step-by-step algorithm was implemented for the investment companies under evaluation so that the reader has a better view of the DEA model with a non-homogeneous indicator and can easily deal with this type. Non-homogeneous indicators in the evaluation of units use the non-homogeneous DEA model. For future work with the idea of implementing the algorithm step by step, we suggest the reader to use this model and the algorithm to rank non-homogeneous DMUs in a way that there exist only one efficient unit with a efficiency value of one and therefore, only one unit has rank one between all units.

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