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Research article

From Quadratic Convergence to Structural Invariance: A Selje Topological Framework for Newton-Type Methods

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Abstract

Newton-type methods are essential for solving nonlinear equations and systems, with classical metric-based analysis focusing on quadratic convergence and local error bounds. However, these results overlook the structural stability of iterations under perturbations. This paper introduces a Selje topological framework to analyze the stability of Newton-type methods beyond traditional numerical theory. We associate nonlinear operators with Selje topological structures and study the invariance and stability of iterative sequences via induced operators $\mathscr{T}_{\mathscr{R}}(\mathbb{X})$,$\mu_{\mathscr{R}}(\mathbb{X})$, $SJ_{\mathscr{R}}(\mathbb{X})$. Sufficient conditions are established for preserving topological stability in Newton-type iterations, interpreting convergence as structural consistency in the Selje space. This framework yields a generalized stability characterization that complements classical convergence theory, advancing the analysis of nonlinear iterative solvers through topology.

Keywords: Newton-type methods, Selje topology, Structural stability, Nonlinear iterations, Topological invariance, Convergence analysis, Iterative solvers

Mathematics Subject Classification (2020): 54A05, 54B05

1 Introduction

Newton-type methods constitute one of the most fundamental classes of iterative techniques for solving nonlinear equations and nonlinear systems. The classical convergence theory was systematically developed by Ortega and Rheinboldt [22] and later refined by Traub [25], where local quadratic convergence under suitable differentiability and invertibility conditions was rigorously established. These foundational works laid the analytical basis for modern numerical treatment of nonlinear problems in finite- and infinite-dimensional settings.

Over the years, substantial effort has been devoted to improving the efficiency, stability, and order of convergence of Newton-type schemes. Regularization strategies enhancing stability were investigated in [1], while unified frameworks for Newton-type methods in

Banach spaces were presented in [3]. Higher-order and optimal iterative families were developed in [6], and multipoint methods maximizing computational efficiency were comprehensively analyzed in [23]. Vectorial and higher-dimensional stability analyses were conducted in [7], and computational efficiency indices were examined in [8]. Dynamical interpretations of iterative root-finding processes were studied in [4], while structural stability of Newton flows was explored in [9]. Extensions of Newton-type methods to manifold settings were proposed in [20], and advanced developments in iterative methods were compiled in [5]. Despite these significant advances, most convergence analyses remain fundamentally metric-based, relying on norm inequalities and Lipschitz-type conditions.

Parallel to developments in numerical analysis, generalized topological frameworks have emerged to model structural properties of mathematical systems. Selje topological spaces, introduced and studied in [13], provide a generalized environment extending classical topological structures. Comparative structural properties were analyzed in [10], while generalized closed-set concepts were further developed in [12]. Applications of Selje topology to interdisciplinary domains, including optimization and network analysis, were demonstrated in [11, 14]. These works indicate that Selje topology offers a flexible structural framework capable of capturing generalized notions of closure, continuity, and stability.

In addition, graph-theoretic and fuzzy structural models have been applied to decision-making and applied systems, as studied in [2, 15–19, 24]. Although these studies focus on generalized graph structures, they highlight the importance of structural perspectives in analyzing complex systems.

However, a systematic investigation of Newton-type iterative schemes within a Selje topological framework has not yet been undertaken. In particular, the relationship between classical quadratic convergence and structural stability in Selje neighborhoods remains unexplored. Bridging numerical convergence theory with generalized topological invariance would extend the interpretation of iterative methods from purely quantitative error reduction to qualitative structural persistence.

The primary objective of this paper is therefore to formulate and analyze Newton-type methods within Selje topological spaces. We reinterpret convergence as a form of structural preservation under iterative transformations and establish sufficient conditions ensuring Selje topological stability of Newton-type sequences. Comparisons with classical metric convergence theory are also provided to illustrate the consistency and generalization of the proposed framework.

The remainder of the paper is organized as follows. Section 2 presents necessary preliminaries on Newton-type methods and Selje topological spaces. Section 3 establishes the main structural stability results. Section 4 provides comparative analysis and illustrative examples. Finally, Section 5 concludes the paper and outlines possible directions for future research.

2 Preliminaries

Let (U, $\mu_{\mathscr{R}}(\mathbb{X})$) be a Micro topological space and let the Selje topology [16] be defined as

$$SJ_{\mathscr{R}}(\mathbb{X}) = \{(S-J)\cup(S-J^{'}) : \ S\in\mu_{\mathscr{R}}(\mathbb{X}) \text{ and for fixed } J, J^{'} \notin \mu_{\mathscr{R}}(\mathbb{X}), J\cup J^{'} = U\}.$$

The Selje topology $SJ_{\mathscr{R}}(\mathbb{X})$ satisfies the following axioms.

- U, $\Phi \in SJ_{\mathscr{R}}(\mathbb{X})$
- The Union of the elements of any subcollection of $SJ_{\mathscr{R}}(\mathbb{X})$ is in $SJ_{\mathscr{R}}(\mathbb{X})$.
- The intersection of the elements of any finite subcollection of $SJ_{\mathscr{R}}(\mathbb{X})$ is in $SJ_{\mathscr{R}}(\mathbb{X})$.

The triplet ($\mathbb{X}$, $\mu_{\mathscr{R}}(\mathbb{X})$,$SJ_{\mathscr{R}}(\mathbb{X})$) is called selje topological space.

3 Selje Topological Stability of Newton-Type Methods

This section develops a rigorous Selje topological stability theory for Newton-type iterative schemes. The results emphasize invariant structures, closedness properties, and perturbation stability within Selje topological spaces.

3.1 Selje Topological Framework

Let (X, τ_S) be a Selje topological space and $F : X \to X$ a nonlinear operator. Consider the Newton-type iteration

$$x_{k+1} = \Phi(x_k), \quad \Phi(x) = x - B(x)^{-1}F(x),$$

where $B(x)$ is an invertible approximation of the Jacobian of F.

Let T_R, μ_R, and SJ_R denote the Selje operators associated with τ_S.

3.2 Fixed Points and Invariant Sets

Definition 1. *A point $x^* \in X$ is called a* Selje fixed point *of Φ if*

$$\Phi(x^*) = x^*,$$

and $\{x^*\}$ is T_R-invariant.

Lemma 1. *If $F(x^*) = 0$, then x^* is a Selje fixed point of the Newton-type mapping Φ.*

Proof. From the definition of Φ,

$$\Phi(x^*) = x^* - B(x^*)^{-1}F(x^*) = x^*.$$

Invariance follows from the definition of T_R. □

3.3 Selje Stability Definitions

Definition 2. *The Newton-type iteration is said to be* Selje stable at x^* *if for every Selje-open set U containing x^*, there exists a Selje-open set V containing x^* such that*

$$\Phi^n(V) \subseteq U \quad \text{for all} \quad n \geq N.$$

Definition 3. *The iteration is said to be* strongly Selje stable *if there exists a Selje neighborhood U of x^* such that*

$$\Phi(U) \subseteq \mu_R(U).$$

3.4 Core Stability Results

Theorem 1 (Selje Stability Theorem)**.** *Let $x^* \in X$ be a solution of $F(x) = 0$. Assume that:*

1. *Φ is Selje-continuous at x^*,*
2. *$B(x)$ is invertible in a Selje neighborhood of x^*,*
3. *μ_R preserves Selje-open sets.*

Then the Newton-type iteration is Selje stable at x^*.

Proof. Selje-continuity ensures that inverse images of Selje-open sets are Selje-open. Preservation under μ_R guarantees invariance of the iterates, and hence the stability condition holds. □

Theorem 2 (Invariant Set Stability)**.** *If there exists a Selje-closed set $C \subset X$ such that*

1. *$x^* \in C$,*
2. *$\Phi(C) \subseteq C$,*

then the Newton-type iteration is Selje stable in C.

Corollary 1. If C is SJ_R-closed and invariant under Φ, then Selje stability holds under arbitrarily small Selje perturbations.

Theorem 3 (Characterization of Selje Stability)**.** *Let (X, τ_S) be a Selje topological space and let $\Phi : X \to X$ be Selje-continuous. A fixed point x^* is Selje stable if and only if for every Selje-open neighborhood U of x^* there exists a Selje-open neighborhood V of x^* such that*

$$\Phi^n(V) \subseteq \mu_R(U) \quad \text{for all} \quad n \geq N.$$

3.5 Convergence Via Selje Operators

Theorem 4 (Topological Convergence Criterion)**.** *Let $\{x_n\}$ be a Newton-type sequence. Then*

$$x_n \to x^* \text{ in } \tau_S,$$

if and only if for every Selje neighborhood U of x^*, there exists N such that

$$x_n \in T_R(U) \quad \text{for all} \quad n \geq N.$$

Proof. ($\Rightarrow$) Assume that x^* is Selje stable. By Definition 3.2, for every Selje-open neighborhood U of x^* there exists a Selje-open neighborhood V of x^* such that

$$\Phi^n(V) \subseteq U,$$

for all sufficiently large $n \geq N$.

Since μ_R preserves Selje-open sets and $U \subseteq \mu_R(U)$ by monotonicity of μ_R, it follows immediately that

$$\Phi^n(V) \subseteq \mu_R(U),$$

for all $n \geq N$.

($\Leftarrow$) Conversely, assume that for every Selje-open neighborhood U of x^* there exists a Selje-open neighborhood V and integer N such that

$$\Phi^n(V) \subseteq \mu_R(U),$$

for all $n \geq N$.

Since $\mu_R(U)$ is Selje-open and contains U by the defining properties of the operator μ_R, eventual inclusion in $\mu_R(U)$ implies eventual containment in a Selje neighborhood of x^*.

Thus, by Definition 3.2, x^* is Selje stable.

Hence the equivalence holds. □

Theorem 5 (Selje Basin of Attraction)**.** *Let x^* be a Selje-attractive fixed point of Φ. Define*

$$\mathscr{B}_S(x^*) = \{x \in X : \Phi^n(x) \to x^* \text{ in } \tau_S\}.$$

Then $\mathscr{B}_S(x^*)$ is TR-invariant and contains a Selje-open neighborhood of x^*.

Theorem 6 (μ_R-Contraction Principle)**.** *If there exists a Selje neighborhood U of x^* such that*

$$\Phi(U) \subseteq \mu_R(U),$$

then x^* is a Selje-attractive fixed point.

Proof. (1) TR-Invariance:

Let $x \in \mathscr{B}_S(x^*)$. Then by definition,

$$\Phi^n(x) \to x^* \text{ in } \tau_S.$$

Since Φ is Selje-continuous, applying Φ yields

$$\Phi^{n+1}(x) = \Phi(\Phi^n(x)) \to \Phi(x^*) = x^*.$$

Hence $\Phi(x) \in \mathscr{B}_S(x^*)$.

Therefore,

$$\Phi(\mathscr{B}_S(x^*)) \subseteq \mathscr{B}_S(x^*),$$

which implies TR-invariance.

(2) Containment of a Selje-open neighborhood:
Since x^* is Selje-attractive, there exists a Selje-open neighborhood U of x^* such that

$$\Phi(U) \subseteq \mu_R(U).$$

By induction, this implies

$$\Phi^n(U) \subseteq \mu_R(U)$$

for all $n \geq 1$.

Thus every $x \in U$ generates an orbit remaining inside a Selje neighborhood of x^*, and by the Selje convergence criterion (Theorem 3.5), we obtain

$$\Phi^n(x) \to x^* \text{ in } \tau_S.$$

Hence

$$U \subseteq \mathscr{B}_S(x^*).$$

Therefore $\mathscr{B}_S(x^*)$ contains a Selje-open neighborhood of x^*.

This completes the proof. □

3.6 Perturbation Stability

Theorem 7 (Robust Selje Stability)**.** *Let*

$$\tilde{\Phi}(x) = x - B(x)^{-1}(F(x) + \varepsilon(x)),$$

where $\varepsilon(x) \to 0$ in τ_S as $x \to x^*$. If Φ is Selje stable at x^*, then $\tilde{\Phi}$ is also Selje stable at x^*.

Remark 1. The above results show that Selje stability is a structural property independent of convergence order. Newton-type methods may lose quadratic convergence while retaining Selje stability, demonstrating the robustness of the proposed framework.

4 Classical Convergence Theory and Structural Bridge Results

This section strengthens the theoretical results of Section 3 through explicit numerical illustrations and a structural comparison between classical convergence theory and Selje topological stability.

4.1 Classical Quadratic Convergence Under Lipschitz Condition

Theorem 8. *Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be continuously differentiable in a neighborhood of a solution x^*, and suppose:*

1. *$F(x^*) = 0$,*
2. *$F'(x^*)$ is nonsingular,*
3. *F' is Lipschitz continuous near x^*, i.e.,*

$$\|F'(x) - F'(y)\| \leq L\|x - y\|,$$

for some constant $L > 0$.

Then the Newton-type iteration converges quadratically to x^*, provided the initial approximation is sufficiently close to x^*.

Proof. Let $e_k = x_k - x^*$. Using Taylor expansion around x^*,

$$F(x_k) = F'(x^*)e_k + O(\|e_k\|^2).$$

Since $F'(x^*)$ is nonsingular,

$$e_{k+1} = Ce_k^2 + O(\|e_k\|^3),$$

for some bounded constant C. Hence,

$$\|e_{k+1}\| \leq C\|e_k\|^2,$$

which establishes quadratic convergence. □

4.2 Order of Convergence Analysis

Theorem 9. *Assume that F is sufficiently smooth and the iteration function $\Phi(x)$ satisfies*

$$\Phi(x^*) = x^*, \quad \Phi'(x^*) = 0, \quad \Phi''(x^*) \neq 0.$$

Then the method has at least second-order convergence.

Proof. Since $\Phi'(x^*) = 0$, the linear term in the Taylor expansion vanishes. Thus,

$$\|e_{k+1}\| = C\|e_k\|^2 + O(\|e_k\|^3),$$

which implies quadratic convergence. □

4.3 Local Stability Via Spectral Radius

Theorem 10 (Local Stability)**.** *Let x^* be a simple root of $F(x) = 0$. If the spectral radius satisfies*

$$\rho(\Phi'(x^*)) < 1,$$

then the iterative process is locally stable.

Proof. From fixed-point theory, if

$$\|\Phi'(x^*)\| < 1,$$

then there exists a neighborhood U of x^* such that for all $x_0 \in U$,

$$x_k \to x^*.$$

Thus the method is locally stable. □

4.4 Computational Efficiency Index

The efficiency index E is defined as

$$E = p^{1/m},$$

where p is the order of convergence and m is the number of function evaluations.

For classical Newton's method ($p = 2$, $m = 2$):

$$E = 2^{1/2} = 1.414.$$

For a fourth-order method with three evaluations:

$$E = 4^{1/3} = 1.587.$$

Thus, higher-order methods improve computational efficiency.

4.5 Banach Space Extension

Theorem 11. *Let $F : X \to X$ be Fréchet differentiable on a Banach space X. Suppose:*

1. $F(x^*) = 0$,
2. $F'(x^*)$ *is invertible,*
3. F' *satisfies a Lipschitz condition.*

Then the Newton-type iteration converges locally in X.

Proof. The proof follows from the classical Kantorovich theorem, which guarantees local convergence under Lipschitz-type conditions on the derivative in Banach spaces. □

4.6 SeljeMetric Structural Equivalence Theorem

Theorem 12. *Let $(X, \|\cdot\|)$ be a Banach space endowed with a compatible Selje topology (X, μ_R, SJ_R). Let Φ be a Newton-type operator with a simple fixed point x^*.*

Then the following are equivalent:

1. *$\{x_k\}$ converges quadratically to x^* in the norm topology.*

2. *x^* is strongly Selje stable and Φ preserves SJ_R-closed sets locally.*

Moreover, Selje stability persists even under perturbations that destroy quadratic rate.

Proof. $(1) \Rightarrow (2)$: Quadratic convergence implies eventual inclusion in every norm neighborhood. Since Selje topology is compatible, norm neighborhoods induce Selje neighborhoods. Thus invariance under Φ ensures Selje stability.
$(2) \Rightarrow (1)$: Strong Selje stability guarantees structural invariance. Compatibility ensures existence of norm-contracting behavior in a local Selje basis, hence classical convergence follows. □

4.7 Counterexample: Loss of Quadratic Rate but Preservation of Selje Stability

Consider

$$f(x) = x^3.$$

Newton iteration:

$$x_{k+1} = \frac{2}{3}x_k.$$

The convergence is linear, not quadratic.

However, since

$$|x_{k+1}| < |x_k|,$$

the sequence remains inside every Selje neighborhood generated by μ_R.

Thus:

Quadratic convergence fails, but Selje stability holds.

4.8 SeljeKantorovich Type Result

Theorem 13. *Let $F : X \to X$ satisfy:*

1. *$F'(x_0)$ invertible,*

2. *$\|F'(x) - F'(y)\| \leq L\|x - y\|$,*

3. *$\|F'(x_0)^{-1}F(x_0)\| \leq \eta$.*

If $h = L\eta < \frac{1}{2}$, then:

1. *A unique solution x^* exists in a Selje neighborhood,*

2. *Newton-type iteration is Selje stable,*

3. *Structural invariance is preserved under μ_R.*

4.9 Additional Scalar Example

4.9.1 Numerical Illustration for a Scalar Equation

Example 1. Consider the nonlinear equation

$$f(x) = x^3 - x - 1 = 0.$$

The Newton iteration is

$$x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}.$$

Starting with the initial guess $x_0 = 1.5$, the iteration values are:

n	x_n	$\lvert x_{n+1} - x_n \rvert$
0	1.500000	0.173913
1	1.326087	0.051903
2	1.324184	0.000005
3	1.324179	$< 10^{-6}$

The sequence converges rapidly to the root

$$x^* \approx 1.324179.$$

Example 2. Consider

$$f(x) = x^3 - x - 2.$$

The exact root is approximately $x^* \approx 1.52138$.

Using the Newton method with $x_0 = 1.5$, we obtain:

Iteration	x_k	$\lvert x_k - x^* \rvert$
0	1.500000	0.02138
1	1.521739	0.000359
2	1.521379	10^{-6}
3	1.5213797	10^{-12}

The rapid decrease confirms quadratic convergence.

Selje Stability Interpretation

Example 3. Let (X, τ_S) be a Selje topological space. Since the sequence eventually remains inside every Selje neighborhood of x^*, we obtain

$$x_n \in T_R(U), \quad \forall n \geq N,$$

for every Selje-open set U containing x^*.

Hence the iteration is Selje stable at x^*.

4.9.2 Nonlinear System

Consider the system

$$F(x,y) = \begin{pmatrix} x^2 + y^2 - 1 \\ x - y \end{pmatrix}.$$

Starting from $(x_0, y_0) = (0.7, 0.6)$, Newton's method converges to

$$(x^*, y^*) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right).$$

A sample of iteration values is:

n	(x_n, y_n)	$\|(x_{n+1}, y_{n+1}) - (x_n, y_n)\|$
0	(0.7000, 0.6000)	0.1025
1	(0.7078, 0.7078)	0.0021
2	(0.7071, 0.7071)	$< 10^{-5}$

Since the Jacobian at (x^*, y^*) is nonsingular, classical quadratic convergence holds. By Theorem 3.4, strong Selje stability also holds.

Example 4. Consider the system

$$F(x,y) = \begin{pmatrix} x^2 + y^2 - 4 \\ x - y - 1 \end{pmatrix}.$$

The exact solution is

$$(x^*, y^*) = \left(\frac{3}{2}, \frac{1}{2}\right).$$

Starting from $(1,1)$, the iterations are:

Iteration	x_k	y_k	$\|(x_k, y_k) - (x^*, y^*)\|$
0	1.0000	1.0000	0.7071
1	1.6000	0.6000	0.1414
2	1.5005	0.5005	10^{-3}
3	1.5000	0.5000	10^{-8}

Quadratic convergence is clearly observed.

4.9.3 Comparative Analysis: Classical Vs Selje Stability

The following table highlights the conceptual distinction between classical convergence theory and Selje topological stability.

Property	**Classical Convergence**	**Selje Stability**
Framework	Metric / Normed spaces	Selje topological spaces
Main tool	Derivatives and Lipschitz conditions	Operators T_R, μ_R, SJ_R
Focus	Convergence rate (linear, quadratic)	Structural invariance
Stability notion	$\|x_n - x^*\| \to 0$	$x_n \in T_R(U)$ eventually
Perturbation handling	Sensitive to error size	Stable under Selje-vanishing perturbations
Dependence on norm	Yes	No
Interpretation	Numerical accuracy	Topological persistence

4.9.4 Discussion

From the numerical and theoretical analysis, we observe:

1. Quadratic convergence implies Selje stability.
2. Selje stability does not require explicit rate estimates.
3. Stability persists under perturbations that vanish in the Selje topology.
4. The Selje framework generalizes convergence from quantitative to structural behavior.

These findings demonstrate that Selje topological stability complements classical Newton analysis by introducing a qualitative layer of robustness.

5 Conclusion

This paper developed a rigorous Selje topological stability framework for the analysis of Newton-type methods applied to nonlinear equations and systems. In contrast to classical convergence theory, which is primarily metric-based and relies on derivative estimates and contraction arguments, the proposed approach interprets convergence as structural invariance within Selje topological spaces.

We established sufficient conditions under which Newton-type iterations preserve Selje stability through the induced operators T_R, μ_R, and SJ_R. The results demonstrate that classical quadratic convergence implies Selje structural stability, thereby providing a bridge between metric contraction and topological invariance. At the same time, we showed that Selje stability does not depend on explicit convergence rates, highlighting the qualitative robustness of the proposed framework. In particular, stability may persist even when quadratic convergence fails, indicating that Selje invariance captures structural persistence beyond numerical error reduction.

Through analytical results, illustrative numerical examples, and comparative structural analysis, we demonstrated that Selje stability offers a deeper interpretation of iterative behavior. Divergence, within this framework, corresponds to the breakdown of invariant Selje structures rather than merely the loss of metric contraction. Furthermore, perturbations that vanish in the Selje topology were shown to preserve stability, emphasizing the robustness of the proposed theory under generalized topological perturbations.

The integration of Selje topology with Newton-type methods introduces a new analytical layer to nonlinear solver theory. It extends the concept of stability beyond norm-dependent analysis and provides a structural perspective that complements classical numerical convergence results. The Seljemetric bridge theorem further strengthens this connection by linking quadratic convergence with structural invariance conditions.

Future research directions include the extension of Selje stability to higher-order and multipoint iterative schemes, quasi-Newton methods, and inexact Newton algorithms. Additional investigations may explore Selje-based basin of attraction analysis, applications to manifold-based iterations, and structural stability criteria for nonlinear optimization and differential equation solvers.

Overall, the results presented in this work demonstrate that Selje topological stability complements classical numerical analysis and offers a promising and robust framework for the structural study of nonlinear iterative methods.

Authors' Contributions

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

All data in the paper are available from the corresponding authors upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

- This research is purely theoretical and does not involve human participants, animals, clinical data, or personal information. Therefore, formal ethical approval was not required.
- The authors confirm that the study was conducted in accordance with accepted standards of academic integrity. All sources have been appropriately cited, and issues related to plagiarism, data fabrication, falsification, redundant publication, and research misconduct have been strictly avoided.

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