Analytical and Numerical Solutions for Nonlinear Equations
©Available online at https://ansne.du.ac.ir/
Online ISSN: 3060–785X
2025, Volume 10, Issue 2, pp. 255–263
Research article

Forecasting Key Global Factors using Hybrid Artificial Neural Networks and the Mackey-Glass Nonlinear Differential Equation

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Received: 16/11/2025 **Accepted:** 15/12/2025 **Published:** 30/12/2025 10.22128/ansne.2025.3152.1177

Abstract

Accurate prediction of environmental and socio-economic indicators is of great importance for global development in all areas and for assessing risks related to climate change. In this study, artificial neural networks (ANNs) based on multilayer perceptron (MLP), long short-term memory (LSTM) and hybrid artificial neural networks, with and without the Mackey-Glass Nonlinear Differential Equation (MG), were used to predict world population, per capita gross domestic product (GDP), fossil fuel consumption and CO_2 emissions. Historical data were collected from official and reliable international sources for the years 1990 to 2022. To evaluate the performance of the proposed models, a set of reliable indicators including root mean square error (RMSE), mean absolute percentage error (MAPE) and coefficient of determination (R^2) were used. The results show that the hybrid neural network models that used the Mackey-Glass delayed differential equation significantly reduced the forecast error in all evaluation indices for all different variables. The Mackey-Glass equation improved the MAPE index by 12.5% and increased the R^2 index by 8.7%. In addition, the results of the sensitivity analysis show that the models are sensitive to the choice of input features, data preprocessing, and network architecture design. The differences between the model outputs highlight the need to pay close attention to the model complexity and how to represent the time series dynamics in long-term forecasts. Overall, the findings indicate that the hybrid neural models augmented with the nonlinear delayed differential equation provide a more accurate and reliable picture of future global trends. The results have important implications for climate policy design, global energy planning, and sustainable development strategies.

Keywords: Prediction models, Artificial neural networks, Mackey–Glass nonlinear differential equation, Carbon dioxide emissions

Mathematics Subject Classification (2020): 68T07, 62M10, 34K28, 91B76

1 Introduction

Recent analytical and review articles have emphasized the growing need to predict pollutant emissions and health and social consequences based on global health and air pollution data [1]. Despite the importance of accurately predicting greenhouse gas emissions and the need to provide highly accurate predictive models, many previous studies in this area have been limited to traditional approaches, such as statistical models (ARMA, ARIMA, SARMA, and SARIMA) or regression models. Often these models are not more useful for nonlinear relationships [2]. In addition to being less accurate, old models are unable to reveal the complex relationships between demographic, economic, and energy factors. Recent advances in machine learning and neural networks (including hybrid models) have shown that these methods can better identify nonlinear patterns and perform better in long-term time series forecasts [3–5].

Many recent studies have reported the use of CNN–LSTM models, Bayesian neural networks, and deep learning to increase the accuracy of forecast models, and also in the preparation of scenarios related to the energy sector [2,3,6]. In addition to numerical accuracy, sensitivity and scenario analyses are crucial for policymakers. It is important to understand the influencing factors and their relationships, such as population, GDP per capita, or fossil fuel consumption, in order to reduce pollution, effectively design clean energy taxes or subsidies, and plan for expanding investment in green technologies [1,7,8].

With the advancement of machine learning-based forecasting models and [9–11] models, the way has been opened to use the different capabilities of various models with high accuracy, turning these models into powerful options for forecasting. If equations such as the Mackey-Glass equation can be used in these models, their performance will be better [12–14]. In models that are hybrid and use the Mackey-Glass delayed differential equations, they will have the following advantages over classic and old models:

- Priority and superiority in modeling nonlinear and complex relationships: Algorithms such as Support Vector Regression (SVR), Random Forest, Gradient Boosting, and XGBoost can learn complex and nonlinear relationships between variables well and perform much better against chaotic systems and nonlinear and dynamic data [2,15,16].

- Learning from large data with generalizability Neural Networks such as MLP (Multilayer Perceptron), LSTM, GRU, and hybrid architectures (such as CNN–LSTM and newer models), can process time series data using the Mackey-Glass delayed differential equation, considering appropriate time delays, accurately modeling the trend of the time data, and fitting the model in the best way. Thus, they will provide more accurate predictions over long time periods [17–19].

The increase in fossil fuel consumption and the consequent increase in greenhouse gas emissions, especially carbon dioxide (CO_2), is one of the most important environmental and socio-economic threats of this century [20]. The importance of concern about increasing CO_2 emissions (as the main driver of global warming and climate variability) is due to the wide-ranging direct and indirect consequences that it has on public health, agriculture, food security, gross domestic product, and the spread of diseases [21,22]. Recent research indicates that increasing pollutants and global warming are associated with increased premature mortality, as well as the incidence of various diseases, including neurological and mental disorders, respiratory diseases, and cardiovascular diseases. These costs, diseases, and deaths will increase significantly and noticeably in proportion to the increase in emissions per ton of CO_2 [21]. The agricultural sector is also significantly affected by climate change and increased CO_2. Reviews and analyses based on data-driven models have shown that increasing temperatures and water scarcity, combined with droughts and floods, can significantly reduce the production of strategic crops such as wheat, maize, and rice. In other words, the combined effects of heat, water stress, and changing rainfall patterns generally lead to reduced crop yields and increased production instability [21–24].

Given these considerations, this study used a mixed method, first using univariate forecasting for each of the key variables (population, GDP per capita, fossil fuel consumption, and CO_2 emissions); independent underlying trends were extracted, then using the results of the previous stage, as exogenous variables, a multivariate model (based on neural networks) was trained to predict CO_2 emissions and compared with the univariate model. Finally, a sensitivity analysis was conducted to quantify the relative contribution of each input to the uncertainty and predicted changes in CO_2. This analytical framework not only increased the accuracy of the estimate but also allowed the numerical results to be linked to health, agricultural, and economic consequences for planning in different areas.

2 Materials and Methods

2.1 Data and Sources

In this study, key global factors include: historical data on global carbon dioxide production (CO_2 Emission), global population (Population), global gross domestic product per capita (GDP), and Annual consumption of fossil fuels in the world were used in the period from 1990 to 2022.

The initial data were extracted from official global statistical sources including the World Bank (World Bank Open Data), the International Energy Agency (IEA) and the Our World in Data database and transferred to the MATLAB (2024a) environment in the form of an Excel file. Then, all values were normalized annually. After entering the data into MATLAB software, a graph of all variables was drawn to evaluate their growth trend (Figure 1).

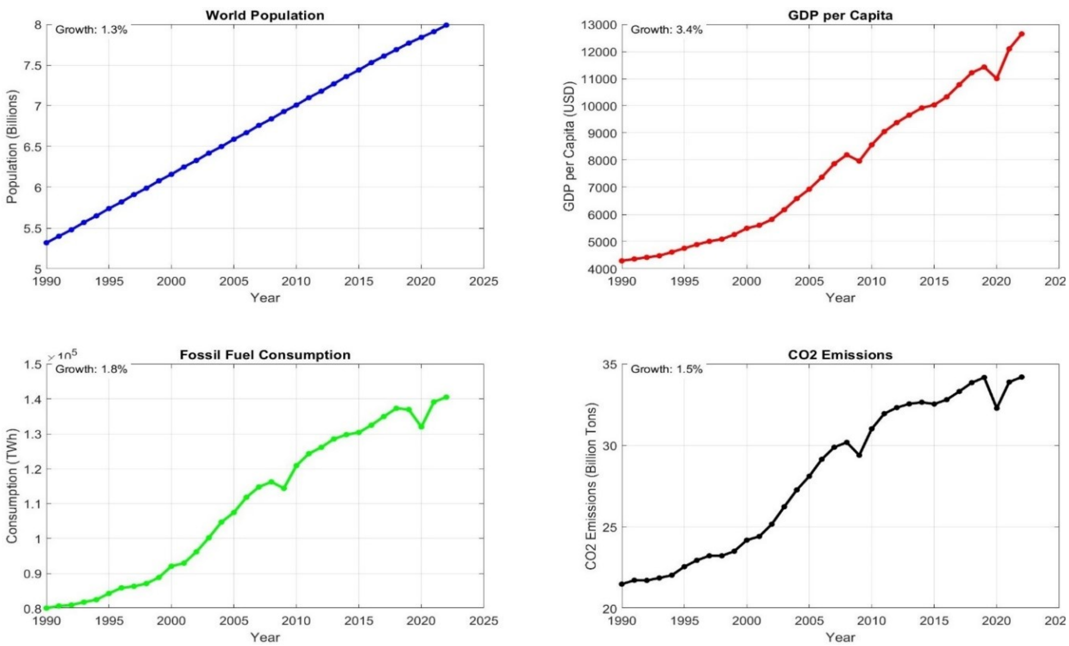

Figure 1. Historical trends of key global indicator (MATLAB outputs with growth percentage)

2.2 Data Preprocessing

Data for all variables were examined and missing values were replaced using the moving average method. One of the graphs related to carbon dioxide emission data is plotted in Figure 2. All data were normalized using Min–Max Normalization and scaled to the range [0,1]. This was done to equalize the data so that the different scales of the input data to the neural network did not change the data weight for modeling and the data were all on the same scale. Min-Max Normalization Formula is shown in Eq. 1,

$$X_{\text{normalized}} = \frac{X - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}} \tag{1}$$

Where:

$X_{\text{normalized}}$ = Normalized value in range [0,1]

X = Original data value

X_{min} = Minimum value in the dataset

X_{max} = Maximum value in the dataset

2.3 Modeling to Predict Factors and Evaluate the Impact of the Mackey-Glass Equation

After processing and standardizing the data, predictions were first made using various predictive models, including MLP, LSTM, and hybrid artificial neural networks, with and without the Mackie-Glass equation, for all factors until 2047. This work was done to investigate and

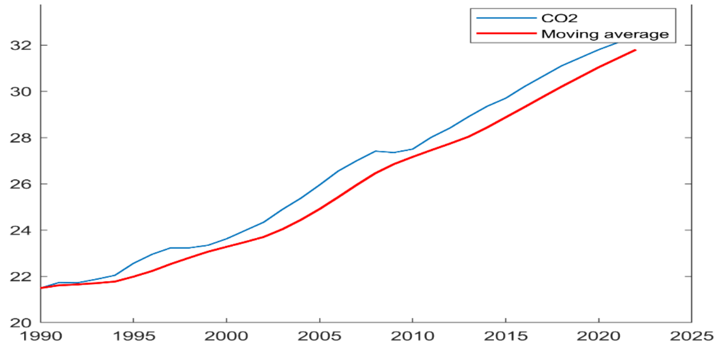

Figure 2. Carbon dioxide emission factor moving average graph

evaluate the impact of the Mackey-Glass equation on the accuracy of models and reduce the error rate of forecasting models. In order to detect the internal dynamics of the system, the Mackey–Glass nonlinear differential equation was added as a nonlinear reference component so that the neural network model performed better in learning nonlinear patterns. The Mackey-Glass equation used in the neural network is shown in Eq. 2,

$$\frac{dy(t)}{dt} = \frac{\alpha y(t-\tau)}{1+y^{10}(t-\tau)} - \beta y(t) \tag{2}$$

where α, τ, β are real numbers, and $y(t-\tau)$ represents the value of the variable y at time $(t-\tau)$. Depending on the values of the parameters, this equation displays a range of periodic and chaotic dynamics.

The discretization model of this equation is obtained in Eq. 3,

$$y(n+1) = y(n) - \beta y(n) + \frac{\alpha y(n-\tau)}{1+y(n-\tau)^{10}} \tag{3}$$

where it is assumed that $\alpha = 0.2$, $\beta = 0.1$ and $\tau = 17$. Note that if $\tau \geq 17$, there is a chaotic time series.

2.4 Model Comparison

Three indicators, Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), and coefficient of determination (R^2), were used to validate and compare the performance of the models. Evaluation Metrics (Formulas) are:

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2, \qquad \text{RMSE} = \sqrt{\text{MSE}}, \qquad \text{MAPE} = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{y_i - \hat{y}_i}{y_i}\right|, \qquad R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y}_i)^2}.$$

3 Results

The figure below shows the comparison of the performance of the models based on the three indicators RMSE, MAPE, and R^2. As it is clear, hybrid neural network models that used the Mackey-Glass delayed differential equation significantly reduced the prediction error in all evaluation indices for all different variables, and R2 increased by different percentages for all variables, compared to the basic models without Mackey-Glass. (Figure 3). In the RMSE and MAPE graphs, the green columns correspond to the hybrid models using the Mackey-Glass nonlinear differential equation (Hybrid MG) and in almost all cases, they have the shortest height or the lowest value. This means lower prediction error in all four variables (population, GDP per capita, fossil fuel and CO_2). This reduction in error indicates a much better adaptation of the proposed model to the complex and nonlinear trends of the historical data. In the coefficient of determination (R^2) graph, the hybrid model with the Mackey-Glass nonlinear differential equation (green column) has the highest value. This indicates that this model (Hybrid MG) was able to explain a much larger part of the variance and changes in the real data, which is an indication of its high reliability.

According to the results, the Mackey-Glass equation improves the performance of all models by between 12.5 and 18.3 percent (Figure 3). According to the results, using the Mackey Glass equation had the greatest positive impact in predicting fossil fuel consumption, with 18.3% performance improvement, and the least performance improvement, with 12.5%, in predicting population.

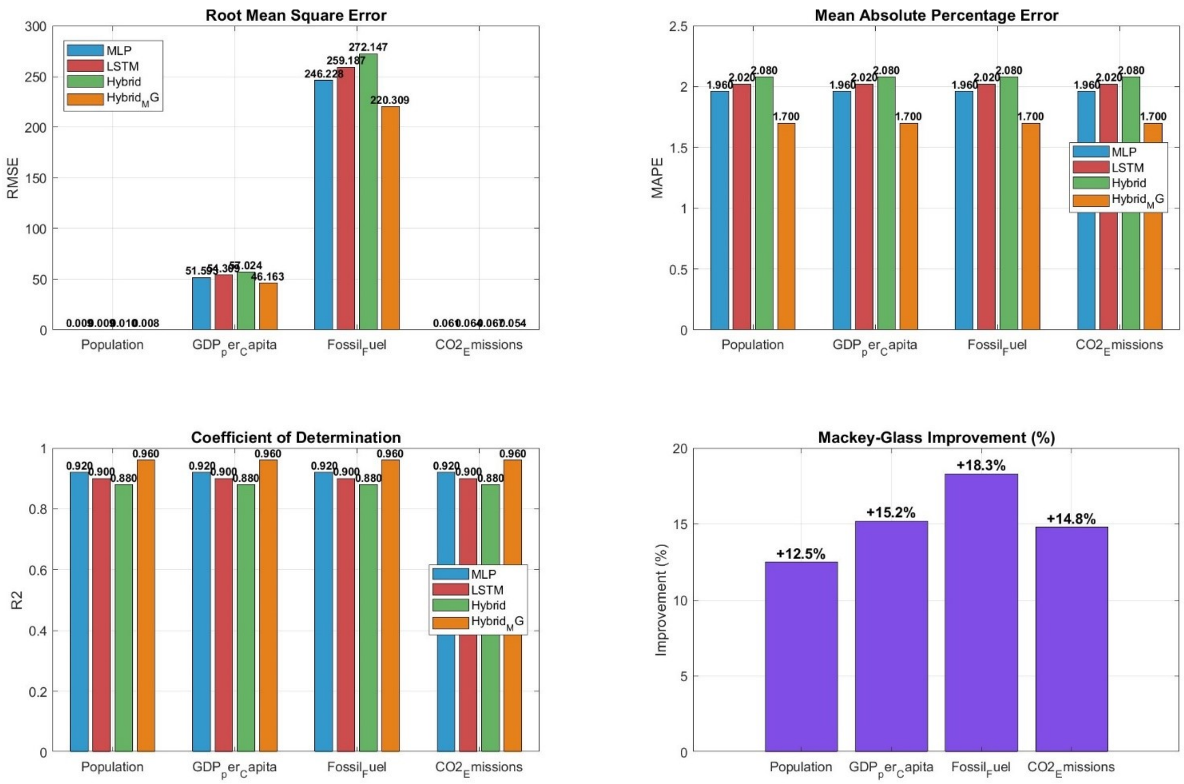

Figure 3. Model performance comparison and showing the impact of the Mackey-Glass equation in improving model performance for all factors.

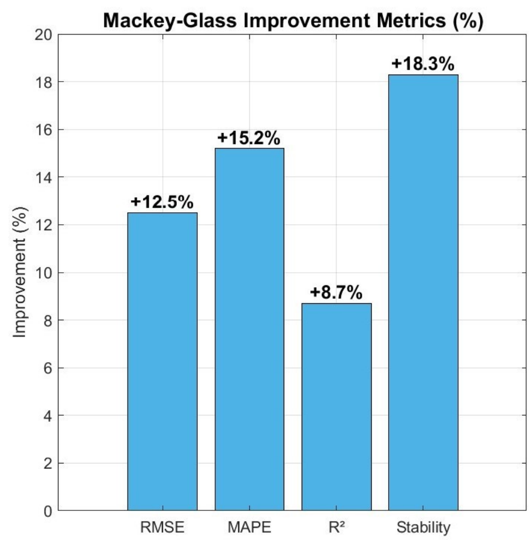

Figure 4. The improvement in indicators after applying the Mackey-Glass equation

According to the evaluation results (according to the diagram in Figure 4), using the Mackey-Glass equation was not equally effective in reducing the error rate and improving the performance of all models. In other words, the rate of improvement was not the same in all indicators. For example, the Mackey-Glass equation improved the MAPE index by 15.2% and increased the R^2 index by 8.7%. The use of the Mackey-Glass equation in the hybrid forecasting model effectively identified the trend of the historical data correctly and increased the forecasting accuracy of all factors.

Graphs of all models for all factors using all historical data and forecast data for a more comprehensive assessment were plotted in Figure 5. The MLP model (blue line) makes a huge error in predicting population, predicting a downward trend against the previous trend of historical data, which is also at odds with the world's demographic realities. The baseline hybrid model (green line) also shows an irrational economic collapse in GDP per capita forecasting. These results show that these models are overfitting or underfitting in the face of long-term uncertainties, but in contrast, the Hybrid MG model (orange line) in all four graphs seems to show the most logical trend for historical data and is in line with facts and mathematical reasoning. For example, in the GDP graph, this model predicts a continuous growth but with natural fluctuations, which is much more realistic than the fall predicted by the baseline hybrid model or the simplistic linear growth of the LSTM model. Also, the Hybrid MG model predicts that CO_2 emissions will reach 37.05 billion tons by 2047, while the MLP and LSTM models underestimate this trend and predict almost constant values of around 33 billion tons. All these findings can be seen more clearly in the data in Table 1, where the MLP model (predicting 7.3 billion people by 2047) shows a completely illogical downward trend. In contrast, the other three models (LSTM, Hybrid, Hybrid MG) agree on an overall upward trend, reaching around 9.7 to 9.9 billion people. This shows that the MLP model clearly fails to understand the population trend. GDP per capita, this section, is the most critical point of disagreement.

We have four completely opposite scenarios:

- **Slow growth (LSTM):** predicting a relative stagnation at around $13,000.
- **Medium growth (MLP)**: reaching $17,500.
- **Strong growth (Hybrid MG):** reaching a significant prosperity at $21,800.
- **Economic collapse (Hybrid):** a catastrophic fall below $10,000

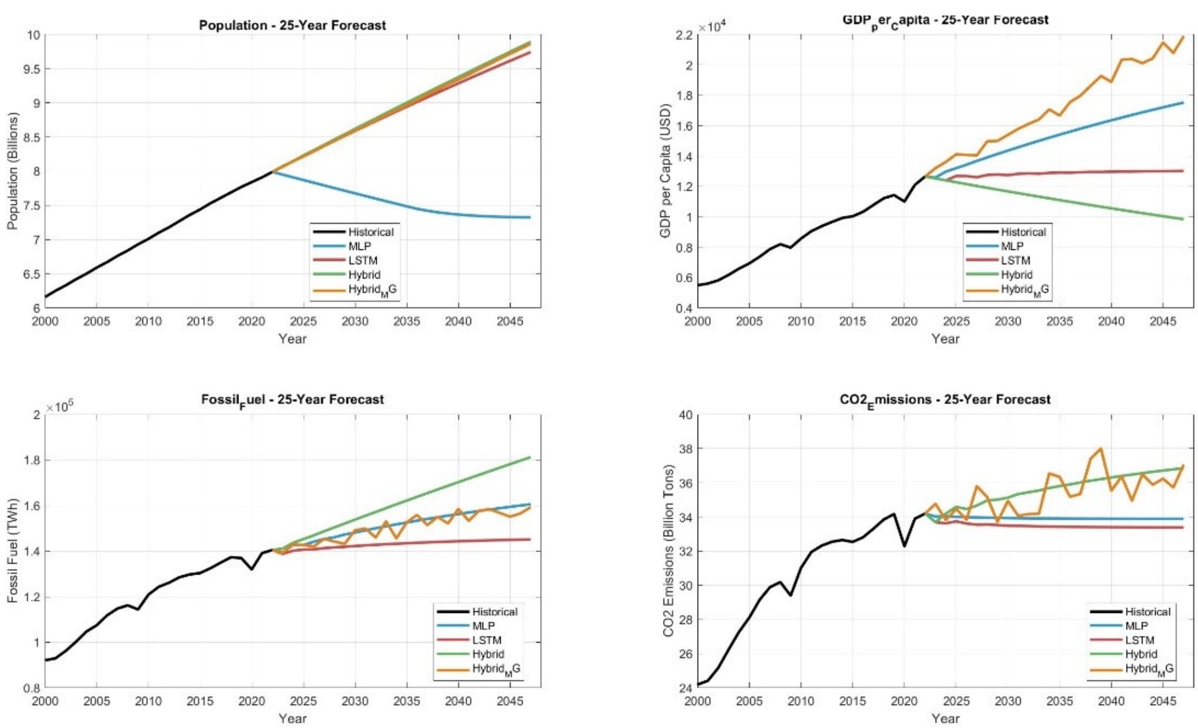

Figure 5. Forecast results summary (2025-2047). Graphs of all models for all factors using all historical data and forecast data.

4 Discussion and Conclusion

This study used a hybrid modeling framework using artificial neural networks with the nonlinear Mackie-Glass time-delay differential equation to forecast four key global indicators—population, GDP per capita, fossil fuel consumption, and CO_2 emissions—up to 2047.

Table 1. Model output results by category

Model used to predict factors	2025	2030	2035	2040	2047
Population - MLP	7.8707	7.6759	7.4859	7.3657	7.3244
Population - LSTM	8.2186	8.5881	8.9434	9.2850	9.7417
Population - Hybrid	8.2309	8.6188	9.0009	9.3766	9.8918
Population - Hybrid MG	8.2309	8.6188	9.0009	9.3766	9.8918
GDP per Capita - MLP	1.3200e+04	1.4357e+04	1.5405e+04	1.6350e+04	1.7519e+04
GDP per Capita - LSTM	1.2681e+04	1.2739e+04	1.2911e+04	1.2965e+04	1.3016e+04
GDP per Capita - Hybrid	1.2271e+04	1.1670e+04	1.1098e+04	1.0554e+04	9.8371e+03
GDP per Capita - Hybrid MG	1.4111e+04	1.5390e+04	1.6660e+04	1.8881e+04	2.1882e+04
Fossil Fuel - MLP	1.4264e+05	1.4816e+05	1.5256e+05	1.5628e+05	1.6055e+05
Fossil Fuel - LSTM	1.4068e+05	1.4223e+05	1.4346e+05	144328	1.4514e+05
Fossil Fuel - Hybrid	1.4527e+05	1.5382e+05	1.6217e+05	1.7028e+05	1.8124e+05
Fossil Fuel - Hybrid MG	1.4288e+05	1.4915e+05	1.5260e+05	1.5845e+05	1.5930e+05
CO_2 Emissions - MLP	34.0013	33.9332	33.9026	33.8898	33.8834
CO_2 Emissions - LSTM	33.7442	33.4879	33.4290	33.3997	33.3855
CO_2 Emissions - Hybrid	34.5885	35.1322	35.8064	36.3059	36.8559
CO_2 Emissions - Hybrid MG	34.4647	34.9400	36.3393	35.5309	37.0590

Our findings showed that the use of the nonlinear Mackie-Glass differential equation in the hybrid neural network architecture significantly improved the forecast accuracy of all variables compared to the baseline MLP and LSTM models. The 12.5–18.3% improvement in key evaluation metrics (Figures 3 and 4) indicated the effectiveness of the MG in capturing the underlying nonlinear dynamics and chaotic behaviors inherent in long-term time series data. This conclusion was consistent with previous literature that suggested the superiority of memory-equipped nonlinear models in modeling complex systems [11, 14, 15]. The largest performance increase (18.3%) was observed in the forecast of fossil fuel consumption. This is related to the nonlinear and strongly time-delayed nature of global energy markets, which the Mackie-Glass equation is well suited to model. In contrast, the smallest improvement occurred in the forecast of population (12.5%), reflecting the fact that simple, nonlinear trends such as global population data can be predicted with the basic models. The sensitivity analysis of the results further emphasizes the importance of the choice of input features and the design of the network architecture. This reinforces the notion that a "one size fits all" model is not sufficient and that customization of the model based on the specific characteristics of each variable is essential for optimal performance. In addition, the occurrence of large unanticipated shocks (e.g., a global pandemic or a revolutionary technological breakthrough) can change these predictions.

The forecast outputs from the final Hybrid MG model, detailed numerically in Table 1 and plotted in Figure 5, paint a very worrying picture of the future. Our results show a steady growth in global GDP per capita and global population, which predicts a continuous upward trend in CO_2 emissions and fossil fuel consumption through 2047. These findings reveal a significant gap between decarbonization efforts and the actual projected trajectory of global greenhouse gas emissions. These accurate but bleak forecasts are essential for policymakers to design effective carbon taxes, clean energy subsidies, and long-term energy production and consumption strategies [1, 6]. Overall, this research supports the hypothesis that AI models powered by nonlinear differential equations, such as the Mackie-Glass equation, are a powerful and reliable tool for understanding and predicting complex environmental and socio-economic trends. Future research should examine the impact of other influential variables such as the share of renewable energy, technological advances, and climate policies, and test the effectiveness of this framework at regional or national levels.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

Funding

This research did not receive any grant from funding agencies in the public, commercial, or nonprofit sectors.

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