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Research article

Some Properties of the Connectivity Index in Vague Graphs with Application

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Abstract

In this paper, we first introduce simple fuzzy graphs (VGs), vague graphs, and then focus on one of the important indices of vague fuzzy graphs, namely the connectivity index, which measures the degree of coordination among the vertices of a graph. We apply this index to study the degree of coordination among the campuses and higher education centers of Farhangian University in Mazandaran Province, which are considered as the nodes of a vague fuzzy graph. The main question is whether these campuses and centers operate as a coordinated network or not. The membership functions of the vertices (nodes) are determined on the basis of the data of the Evaluation and Supervision Office of Farhangian University in Mazandaran Province, obtained mainly from student questionnaires in different domains such as the presidents office, administration, finance, research, cultural affairs and student services. At the end of the paper we conclude that the campuses and centers of Farhangian University do not yet behave as a perfectly coordinated network; however, by implementing the changes and improvements suggested in this paper they can become fully coordinated. Likewise, some new indices such as Zagreb index, sombor index, wiener index, and average wiener index are introduced.

Keywords: Vague fuzzy graph, Connectivity index, Application

Mathematics Subject Classification (2020): 05C99, 05C78, 03E72

1 Introduction

Graph theory has a rich history dating back to the 18th century. Its origins go back to the famous Königsberg bridge problem, which was solved by the Swiss mathematician Euler in 1736. Eulers solution to this problem laid the foundation for the development of graph theory as a distinct area of mathematics. Graphs are mathematical structures used to model relationships and connections between objects. They are made of vertices (nodes) and edges (links). Graphs represent many real-world networks such as social networks, computer networks, transportation systems and biological networks.

In 1965, Zadeh [22] introduced fuzzy sets as an expansion of crisp sets, aiming to address ambiguity and handle uncertain information. Vague set was the very fast generalization of fuzzy sets which was introduced by Gau [5]. A fuzzy graph extends classical graphs by allowing uncertainty and partial membership. In a fuzzy graph, nodes and edges can have membership values between 0 and 1, indicating how strongly they belong. Also, fuzzy graphs serve as a powerful tool in decision-making by introducing the ability to model uncertainty, relationships, and imprecision in complex systems. They are particularly useful when dealing with scenarios where the available information is not exact or crisp, allowing decision-makers to arrive at more realistic, informed, and flexible decision in uncertain environments.

Rosenfeld described the term fuzzy graphs [17]. Bhattacharya [3] discussed the relationship between fuzzy groups and fuzzy graphs. Gani and Latha [6] studied irregularity of fuzzy graphs and the degree and size of fuzzy graphs were calculated by Gani and Ahmad [7].

Vague graphs was defined in [11]. Rao et al. [12, 13] studied domination set and equitable domination set in vague graphs. Borzooei and Rashmanlou [1, 2] defined new concepts relating to VGs, product of VGs [15], and vague competition graphs [2]. Talebi et al. [18–20] defined isomorphism and some operations on VGs. Kosari et al. [9] investigated topological indices in fuzzy graphs. Wiener [21] Proposed Wiener index. Gutman and Trinajsti [8] defined Zagreb index. New results of the second Zagreb index studied by Das and Gutman [8]. Poulik and Ganesh [10] presented Randic index of bipolar fuzzy graphs. In this paper, we defined the important indices of vague graphs, namely first Zagreb index, second Zagreb index, Hyper Zagreb index and Sombor index, wiener index, and new results are introduced. Also an application of the connectivity index to Farhangian university of Mazandaran is given.

2 Preliminaries

- **Fuzzy set.** Every (crisp) set is characterized by a property that determines membership of its elements. This property is represented by a membership function. If the set is denoted by A, then its membership function μA is a map $\mu A : X \to \{0,1\}$ on a universe X, where $\mu A(x) = 1$ if x belongs to A and $\mu A(x) = 0$ otherwise. A fuzzy set generalizes this notion by allowing $\mu A(x)$ to take any value in the interval $[0,1]$. A fuzzy set A on X can be written as the set of ordered pairs $\{(x, \mu A(x)) \mid x \in X\}$.

- **Simple fuzzy graph.** Let V be a non-empty set. A fuzzy graph $\tilde{G}$ on V is an ordered pair (σ, μ) where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ Equivalently, $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ satisfy $\mu(u,v) \le \sigma(\mu) \wedge \sigma(v)$ for all $u, v \in V$. The underlying crisp graph $G : (V,E)$ is the graph used in classical discrete mathematics. In our notation we write $\tilde{G} = (\sigma, \mu; G)$. (See Figure 1)

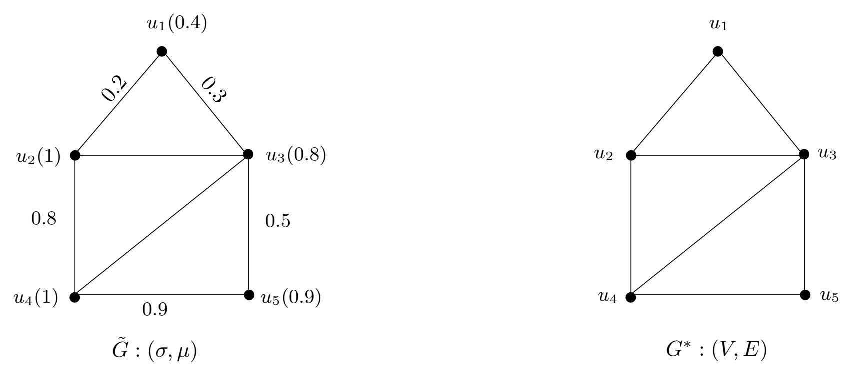


Figure 1. Fuzzy graph $\tilde{G}$ and its crisp graph

- **Vague set.** A vague set A on a finite non-empty set X finite non-empty set (t_A, f_A), where $t_A : X \to [0,1]$ and $f_A : X \to [0,1]$ are respectively the truth-membership and falsity-membership functions such that for every $x \in X$ we have $0 \le t_A(x) + f_A(x) \le 1$.

- **Vague relation.** Let X and Y be finite non-empty sets. A vague relation R from X to Y is a vague set on $X \times Y$, that is

$$R = \{\langle (x,y), t_R(x,y), f_R(x,y) \rangle \mid x \in X, y \in Y\},$$

where $t_R : X \times Y \to [0,1]$ and $f_R : X \times Y \to [0,1]$ satisfy $0 \le t_R(x,y) + f_R(x,y) \le 1$ for all $(x,y) \in X \times Y$.

- **Vague graph.** Let $G^* = (V,E)$ be a (crisp) graph. An ordered pair (A,B) on G^* is called a vague graph on G^* if $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that for every edge $xy \in E$ the following conditions hold:

$$t_B(xy) \le \min(t_A(x), t_A(y)), \quad f_B(xy) \ge \max(f_A(x), f_A(y)).\text{(See Figure2)}$$

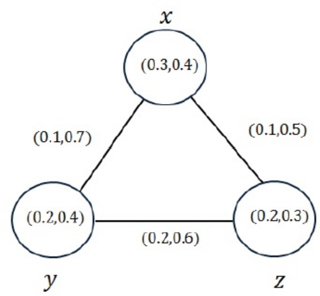


Figure 2. Vague graph G

- **Path.** A path in a vague graph G is a sequence of distinct vertices $a_1a_2\cdots a_n$ such that for some i and j one of the following holds:
  1) $t_B(a_i,a_j) > 0, \quad f_B(a_i,a_j) = 0,$
  2) $t_B(a_i,a_j) = 0, \quad f_B(a_i,a_j) > 0,$
  3) $t_B(a_i,a_j) > 0, \quad f_B(a_i,a_j) > 0.$

- **t-strength.** The t-strength of a path between two vertices is the minimum of the truth-membership values of its edges. For vertices a_i and a_j we write
$$S_t = \min\{t_B(a_i,a_j)\}.$$

- **f-strength.** The f-strength of a path between two vertices is the maximum of the falsity-membership values of its edges. For vertices a_i and a_j we write
$$S_f = \max\{f_B(a_i,a_j)\}.$$

- **Connectivity index.** For vertices a and b in a vague fuzzy graph G we define:
$$Conn_t(G)(a,b) = \max S_t(a,b), \qquad Conn_f(G)(a,b) = \min S_f(a,b),$$
where the maxima and minima are taken over all paths between a and b. The connectivity index of G is then defined as:
$$CI(G) = CI_t(G) + CI_f(G),$$
where
$$CI_t(G) = \sum t_A(a)\cdot t_A(b)\cdot Conn_t(G)(ab),$$
$$CI_f(G) = \sum f_A(a)\cdot f_A(b)\cdot Conn_f(G)(ab),$$
the sums being taken over all unordered pairs of distinct vertices a, b.

Example 1. Consider the vague graph G (See Figure 3) with three vertices a, m and d. The corresponding membership values (t_A, f_A) shown in the original paper. By computing the t-strength and f-strength of the paths between the vertices t-strength:
$$S_t(am) = \min\{t_B(am)\} = \min\{0.5,0.7\} = 0.5, \quad S_t(ad) = S_t(md) = 0.5,$$
f-strength:
$$S_f(am) = \max\{f_B(am)\} = \max\{0.3,0.2\} = 0.3, \quad S_f(ad) = S_f(md) = 0.3,$$
we obtain $CI_t(G) = 0.73$, $CI_f(G) = 0.024$, and therefore $CI(G) = 0.754$.

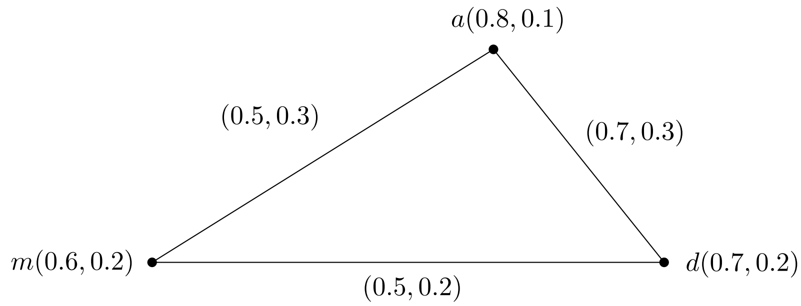


Figure 3. Vague graph G

3 Main ResuLts

Definition 1. *The first Zagreb index of a vague graph $G=(M,N)$ of the graph $G^*=(V,E)$ is denoted by:*

$$ZF^1_{VG}(G)=\left(ZF^{1\theta}_{VG}(G),ZF^{1\mu}_{VG}(G)\right)=\left(\sum_{p\in V(G)}[\theta_M(p)d^\theta(p)]^2,\sum_{p\in V(G)}[\mu_M(p)d^\mu(p)]^2\right),$$

where the true and false parts of the degree of the node p in G are denoted by $d^\theta(p)$ and $d^\mu(p)$, respectively.

Definition 2. *A VG $G=(M,N)$ of a graph $G^*=(V,E)$ exhibits a second Zagreb index, $ZF^2_{VG}(G)$, which is defined as follows:*

$$ZF^2_{VG}(G)=\left(ZF^{2\theta}_{VG}(G),ZF^{2\mu}_{VG}(G)\right)=\Big(\sum_{1\neq j,p_ip_j\in E(G)}\theta_m(p_i)\theta_m(p_j)d^\theta(p_i)d^\theta(p_j),\sum_{i\neq j,p_ip_j\in E(G)}\mu_m(p_i)\mu_m(p_j)d^\mu(p_i)d^\mu(p_j)\Big),$$

where the true and false parts of the node p_i's degree are denoted respectively by $d^A(p_i)$ and $d^\mu(p_i)$.

Definition 3. *For the graph $G^*=(V,E)$, the Hyper Zagreb index of a VG, $G=(M,N)$ is $HZI_{VG}(G)$. It is defined as:*

$$HZI^2_{VG}(G)=\left(HZI^\theta_{VG}(G),HZI^2_{VG}(G)\right)=$$
$$\Big(\sum_{i\neq j,p_ip_j\in E(G)}[\theta_m(p_i)d^\theta(p_i)+\theta_m(p_j)d^\theta(p_j)]^2,\sum_{i\neq j,p_ip_j\in E(G)}[\mu_m(p_i)d^\mu(p_i)+\mu_m(p_j)d^\mu(p_j)]^2\Big),$$

where the degree of the node p_i in G are represented by the numbers $d^\theta(p_i)$ and $d^\mu(p_i)$, respectively.

Definition 4. *The Sombor index of a VG denoted as $G=(M,N)$ derived from the graph $G^*=(V,E)$ is symbolized as $S\circ F_{VG}(G)$ and is defined as follows:*

$$S\circ F_{VG}(G)=\left(S\circ F^\theta_{VG}(G),S\circ F^\mu_{VG}(G)\right)=$$
$$\left(\sum_{i\neq j,p_ip_j\in E(G)}\sqrt{\{\theta_m(p_i)d^\theta(p_i)\}^2+\{\theta_m(p_j)d^\theta(p_j)\}^2},\sum_{i\neq j,p_ip_j\in E(G)}\sqrt{\{\mu_m(p_i)d^\mu(p_i)\}^2+\{\mu_m(p_j)d^\mu(p_j)\}^2}\right).$$

Theorem 1. *Let $G=(M,N)$ be a connected VG and $G'=(M',N')$ such that $V'=V-\{p_n\}$, $p_n\in V$ with $O(V)=n$. Then, $S\circ F^\theta_{VG}(G)\geq S\circ F^\theta_{VG}(G')$ and $S\circ F^\mu_{VG}(G)\geq S\circ F^\mu_{VG}(G')$.*

Proof. Since the nodes of G are n, let $V=\{p_1,p_2,\cdots,p_n\}$ and $V'=\{p_1,p_2,\cdots,p_n\}$. So, G' is a vague sub-graph of G since V' must be a subset of V. Hence, $\theta_M(p_i)=\theta_{M'}(p_i)$, $\mu_M(p_i)=\mu_{M'}(p_i)$, $\forall\ p_i\in V'$ and $\theta_N(p_ip_j)=\theta_{N'}(p_ip_j)$, $\mu_N(p_ip_j)=\mu_{N'}(p_ip_j)$, $\forall\ p_ip_j\in E'$. Now the expression

$$d^\theta(p_i)=\sum_{p_ip_j\in E}\theta_N(p_ip_j),$$

where

$$d^\mu(p_i)=\sum_{p_ip_j\in E}\mu_N(p_ip_j),$$

or $d^{\theta}(p_i)$ and $d^{\mu}(p_i)$ represented the sum of the membership values of the edges incident in p_i in G, whose values are true and false, respectively. Also,

$$d'^{\theta}(p_i) = \sum_{p_ip_j \in E'} \theta_N(p_ip_j), \qquad \text{and} \qquad d'^{\mu}(p_i) = \sum_{p_ip_j \in E'} \mu_N(p_ip_j),$$

i.e. $d'^{\theta}(p_i)$ and $d'^{\mu}(p_i)$ are respectively sum of the true and false membership values of the edges incident in p_i in G'. So,

$$\sqrt{\{\theta_M(p_i)d^{\theta}(p_i)\}^2 + \{\theta_M(p_j)d^{\theta}(p_j)\}^2} \geq 0, \qquad \sqrt{\{\theta_M(p_i')d'^{\theta}(p_i)\}^2 + \{\theta_M(p_j')d'^{\theta}(p_j)\}^2} \geq 0,$$
$$\sqrt{\{\mu_M(p_i)d^{\mu}(p_i)\}^2 + \{\mu_M(p_j)d^{\mu}(p_j)\}^2} \geq 0, \qquad \sqrt{\{\mu_M(p_i')d'^{\mu}(p_i)\}^2 + \{\mu_M(p_j')d'^{\mu}(p_j)\}^2} \geq 0.$$

So, we conclude that

$$\sum_{\substack{1 \leq i \neq j \leq n \\ p_ip_j \in E(G)}} \sqrt{\{\mu_M(p_i')d'^{\mu}(p_i)\}^2 + \{\mu_M(p_j')d'^{\mu}(p_j)\}^2} \geq 0.$$

So,

$$S \circ F^{\theta}_{VG}(G) \geq S \circ F^{\theta}_{VG}(G'), \qquad \text{and} \qquad S \circ F^{\mu}_{VG}(G) \geq S \circ F^{\mu}_{VG}(G').$$

□

Definition 5. *The wiener index of a VG $G = (M,N)$ is defined as:*

$$\begin{aligned}
WI(G) &= \sum_{a,b \in M^*} \theta_M(a)\mu_M(b)\theta_M(b)\mu_M(b)L(a,b) \\
&= \sum_{a,b \in M^*} \theta_M(a)\mu_M(b)\theta_M(b)\mu_M(b)(L_{\theta}(a,b), L_{\mu}(a,b)) \\
&= \sum_{a,b \in M^*} [\theta_M(a)\theta_M(b)L_{\theta}(a,b) + \mu_M(a)\mu_M(b)L_{\mu}(a,b)] \\
&= \sum_{a,b \in M^*} [\theta_M(a)\theta_M(b)L_{\theta}(a,b) + \sum_{a,b \in M^*} \mu_M(a)\mu_M(b)L_{\mu}(a,b)] \\
WI(G) &= \theta WI(G) + \mu WI(G).
\end{aligned}$$

Example 2. Consider the VG G as Figure 4 we have:

$$\theta WI(G) = 0.063 + 0.012 + 0.024 + 0.016 = 0.115,$$
$$\mu WI(G) = 0.18 + 0.088 + 0.24 + 0.011 = 0.618.$$

So, $WI(G) = 0.115 + 0.618 = 0.733$.

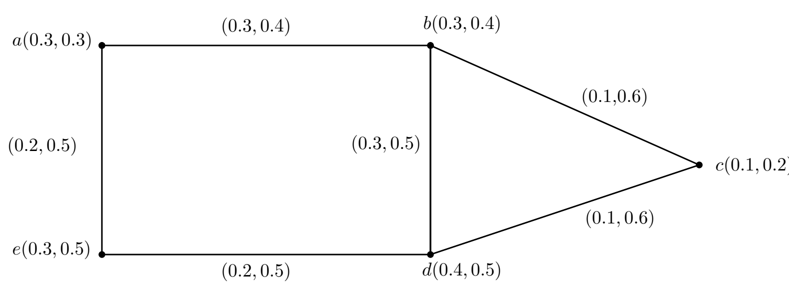


Figure 4. Vague graph G

Definition 6. *Let $G = (M,N)$ be a VG. The average $\theta - WI$ is defined as:*

$$A\theta WI(G) = \frac{1}{\binom{n}{2}} \left(\sum_{(a,b) \in N^*} \theta_M(a)\theta_M(b)L_{\theta}(a,b) \right),$$

and the AV $\mu-WI$ G is defined as:

$$A\mu WI(G)=\frac{1}{\binom{n}{2}}\left(\sum_{(a,b)\in N^*}\mu_M(a)\mu_M(b)L_\mu(a,b)\right).$$

Definition 7. *Let G be a VG. The AWI of G is defined as sum of AV θ-WI and AV μ-WI of G.*

$$AWI(G)=\frac{1}{\binom{n}{2}}\left(\sum_{(a,b)\in N^*}\theta_M(a)\theta_M(b)L_\theta(a,b)\right)+\frac{1}{\binom{n}{2}}\left(\sum_{(a,b)\in N^*}\mu_M(a)\mu_M(b)L_\mu(a,b)\right).$$

$$AWI(G)=A\theta WI(G)+A\mu WI(G).$$

Example 3. Consider the VG G as Figure 5. For all nodes $(\theta_M,\mu_M)=(0,8,0.1)$. The WI for G is given as:

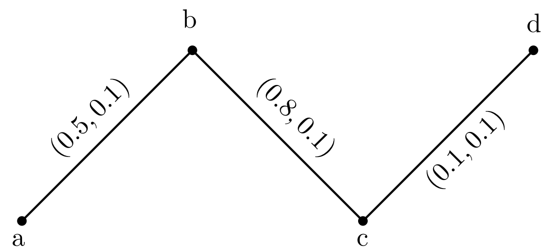


Figure 5. Vague graph G

$$\begin{aligned}\theta WI(G) &= (0.8)(0.8)(0.5)+(0.8)(0.8)(0.8)+(0.8)(0.8)(0.1)\\ &\quad +(0.8)(0.8)(1.3)+(0.8)(0.8)(0.9)+(0.8)(0.8)(1.4)=3.2.\\ A\theta WI(G) &= \frac{1}{\binom{4}{2}}(\theta WIG)=\frac{1}{6}(3.2)=0.53.\\ \mu WI(G) &= (0.1)(0.1)(0.1)+(0.1)(0.1)(0.1)+(0.1)(0.1)(0.1)\\ &\quad +(0.1)(0.1)(0.2)+(0.1)(0.1)(0.2)+(0.1)(0.1)(0.3)=0.01,\\ A\mu WI &= \frac{1}{\binom{4}{2}}(\mu WIG)=\frac{1}{6}(0.01)=0.0017.\end{aligned}$$

So, $WI(G)=3.2+0.01=3.21$, and $AWIG=0.53+0.0017=0.5317$.

Proposition 1. *Let $G=(M,N)$ be a VG that has only $\theta-$strong arcs i.e.,*

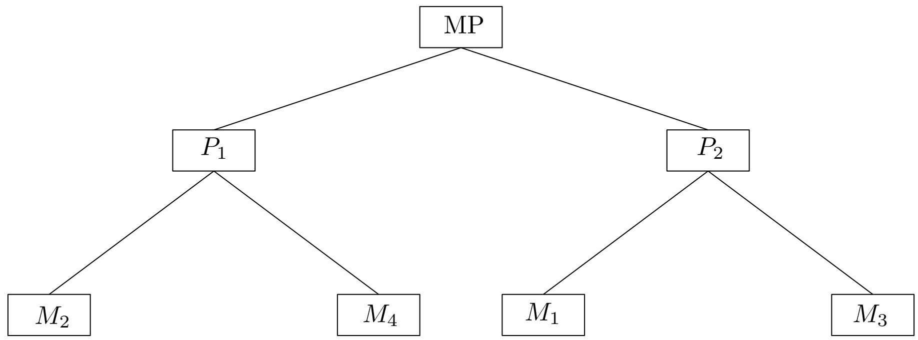
$$\theta_N(a_i,a_j)=CONN_{\theta(G)-(a_i,a_j)}(a_i,a_j),\qquad and \qquad \mu_N(a_i,a_j)=CONN_{\mu(G)-(a_i,a_j)}(a_i,a_j),$$

then $CI(G)=WI(G)$.

Proof. Let $G=(M,N)$ be a VG with only θ-strong edges and each node is connected with every other node with the help of an arc. For each two pair of nodes a,b, $\theta_N(a,b)=CONN_{\theta(G)-(a,b)}(a,b)=L_\theta(a,b)$ and $\mu_N(a,b)=CONN_{\mu(G)-(a,b)}(a,b)=L_\mu(a,b)$.
So, $CI(G)=WI(G)$. □

3.1 Application of the Connectivity Index in Farhangian University of Mazandaran

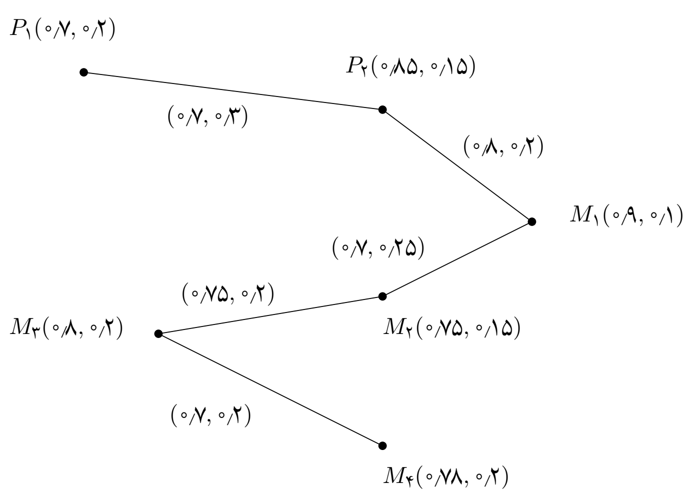
Farhangian University of Mazandaran consists of two campuses and four higher education centers. Campus P_1 is the male campus in Sari, with two affiliated centers the male center in Babol (M_2) and the center in Nowshahr (M_4). Campus P_2 is the female campus in Sari, with



two affiliated centers: the female center in Qaemshahr (M_1) and the center in Amol (M_3). The two campuses are managed by a single administrative unit called the Management of Campuses of Farhangian University of Mazandaran; that is, all campuses and centers of the province are governed under a unified management. Our investigation focuses on these two campuses and four centers, all of which currently host students. The performance of the campuses and centers is measured using the results of student evaluation surveys conducted by the provincial Evaluation and Supervision Office in various domains such as education, welfare (dormitory, catering), cultural activities, student affairs. The detailed forms are presented in the appendix of the original Persian version. Using these data we construct a vague fuzzy graph whose vertices correspond to the two campuses and four centers, and whose edges represent their connections. The truth-membership and falsity-membership functions for the vertices are derived from the performance percentages reported for each unit.

In this model the vague fuzzy graph has six vertices P_1, P_2, M_1, M_2, M_3 and M_4. The approximate values of the vertex memberships, obtained from the evaluation data, are as follows:

$$\begin{aligned} t_A(P_1) &= 0.70, \quad f_A(P_1) = 0.20, \\ t_A(P_2) &= 0.85, \quad f_A(P_2) = 0.15, \\ t_A(M_1) &= 0.90, \quad f_A(M_1) = 0.10, \\ t_A(M_2) &= 0.75, \quad f_A(M_2) = 0.15, \\ t_A(M_3) &= 0.80, \quad f_A(M_3) = 0.20, \\ t_A(M_4) &= 0.78, \quad f_A(M_4) = 0.20. \end{aligned}$$



The membership values assigned to the edges are chosen in accordance with the vague fuzzy graph conditions $t_B(xy) \leq \min\{t_A(x), t_A(y)\}$ and $f_B(xy) \geq \max\{f_A(x), f_A(y)\}$. Because the male campus in Sari and the Babol center together host the largest number of students in the province, a %10 uncertainty was assumed in their data, which is reflected in the membership values.

Using the definitions of tstrength and fstrength given in Section 2, we compute the tstrength and fstrength of the paths between P_1 and each of the other five vertices. For example,

$$\begin{aligned} S_t(P_1P_2) &= 0.7, \\ S_t(P_1M_1) &= \min\{0.7, 0.8\} = 0.7, \\ S_t(P_1M_2) &= \min\{0.7, 0.8, 0.7\} = 0.7, \end{aligned}$$

and similarly for M_3 and M_4. Likewise, the corresponding fstrengths are:

$$\begin{aligned} S_f(P_1P_2) &= 0.3, \\ S_f(P_1M_1) &= \max\{0.3, 0.2\} = 0.3, \\ S_f(P_1M_2) &= \max\{0.3, 0.2, 0.25\} = 0.3, \end{aligned}$$

and so on. From these values we obtain the connectivity contributions

$$\begin{aligned} &Conn_t(P_1P_2) = 0.7, \\ &Conn_f(P_1P_2) = 0.3, \\ &Conn_t(P_1M_1) = 0.7, \\ &Conn_f(S)(P_1M_1) = 0.3, \qquad (i = 1, 2, 3, 4), \\ &CI(G) = \sum t_A(a) \cdot t_A(b) \cdot Conn_{t(S)}(ab) + \sum f_A(a) \cdot f_A(b) \cdot Conn_{f(S)}(ab), \\ &CI_{t(G)} = t_A(P_1) \cdot t_A(P_2) \cdot Conn_{t(S)}(P_1P_2) + t_A(P_1) \cdot t_A(M_1) \cdot Conn_{t(S)}(P_1M_1) + t_A(P_1) \cdot t_A(M_2) \cdot Conn_{t(S)}(P_1M_2) \\ &\qquad + t_A(P_1) \cdot t_A(M_3) \cdot Conn_{t(S)}(P_1M_3) + t_A(P_1) \cdot t_A(M_4) \cdot Conn_{t(S)}(P_1M_4) \\ &\qquad = 0.7 \times 0.8 \times 0.7 + 0.7 \times 0.9 \times 0.7 + 0.7 \times 0.75 \times 0.7 + 0.7 \times 0.8 \times 0.7 + 0.7 \times 0.78 \times 0.7 \\ &\qquad = 0.392 + 0.441 + 0.368 + 0.392 + 0.382 = 1.975. \end{aligned}$$

Substituting these numbers into the formula for the connectivity index yields:

$$CI_t(G) = 1.975, \quad CI_f(G) = 0.048,$$

and therefore the total connectivity index of the vague fuzzy graph representing the six units of Farhangian University of Mazandaran is

$$CI(G) = CI_t(G) + CI_f(G) = 2.023.$$

So, $CI_f(G) = 0.048$.

3.2 Conclusion

The connectivity index of a vague fuzzy graph is a suitable tool for investigating the relationships, performance and networklike behaviour of an organization such as a university, a governmental office, or even smaller units such as a family or a school. It indicates to what extent the members of a group operate in a coordinated way.

Applying this index to the campuses and centers of Farhangian University of Mazandaran, we observed that there is not yet complete coordination among them, although their performance levels are relatively close. If decisions are made and implemented in a genuinely provincial mannerwithout excessive dependence on the personal preferences of local managersand if the decisions of the provincial management are executed consistently in all campuses and centers, the degree of cooperation and coordination among these units will increase significantly.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

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