Analytical and Numerical Solutions
for Nonlinear Equations
©Available online at https://ansne.du.ac.ir/
Online ISSN: 3060–785X
2024, Volume 9, Issue 2, pp. 221–252
Research article

Optimizing of the Used Commodities Recycling Process Problem by Designing a Reverse Logistics Network using the Multi-Objective Programming Method

Gholam Hassan Shirdel [1,*], Ali Abdoljabbar Fadhil[1]

[1] Department of Mathematics, Faculty of Sciences, University of Qom, Qom, Iran

* **Corresponding author(s):** g.h.shirdel@qom.ac.ir

Received: 29/06/2025 **Revised:** 07/09/2025 **Accepted:** 16/09/2025 **Published:** 07/10/2025 10.22128/ansne.2025.1037.1140

Abstract

The increase in urban population, change in lifestyle, change in diet, as well as increase in the level of well-being and living standards in urban communities have caused a large amount of solid waste in big cities. Currently, managing the process of solid waste in big cities is one of the most important problems in developing countries. Most of the studies in the literature are focused on reverse logistics for one type of product for the recovery or recycling process, and not much attention has been paid to the reuse distribution network through charities. In this research, a framework for reusing all kinds of household appliances to reduce urban solid waste and help low-income families is proposed. A mixed integer linear mathematical model with uncertainty in the number of products is presented for reverse logistics network optimization. This model has been solved by used Genetic Algorithm and its applications have been discussed. In the designed logistics network, various topics such as recycling, repair and charity centers are considered. In order to show the performance of the presented model, a numerical example has been solved by software MATLAB. In this example, the software MATLAB obtains the network structure at an optimal cost. The results confirm the applicability of the model by providing a large number of second-hand products that can be transported and reused at an affordable cost.

Keywords: Logistics network, reverse logistics, recycling, waste, reduce waste, charities, mixed integer linear mathematical model

Mathematics Subject Classification (2020): 90B06, 90B18, 90B50, 90C06, 90C11, 90C17, 90C29, 90C35, 90C90.

1 Introduction

Waste is a product that the original owners no longer need, which will either become useless or be thrown away. Improper use and inefficient management of waste causes environmental contamination, disease transmission, high energy consumption per capita, global warming, and more importantly, environmental instability (the cycle of returning materials to nature). Municipal solid waste is a direct and inevitable result of urbanization and development and is considered a real environmental threat [16]. In addition, rapid urbanization, economic boom and industrialization have caused excessive waste generation in the past decades [7]. Managing the increasing volume of household waste in developing Asian countries has become a difficult challenge in recent years [13].

The circular economy, which is part of sustainable development, inspires waste reduction [14]. Meanwhile, the sustainable implementation of supply chain management can help ensure long-term environmental, social and economic benefits. For this reason, it is necessary to design and develop planning networks and systems for municipalities and sustainable waste organizations under the influence of this issue.

There are various definitions of reuse and other related concepts in the literature, classified by Amiri et al [3]. We refer to the definition of the European Commission in 2010 [14]. Reuse means any operation that provides the process of exploitation of good used goods or their used components that reach the end of their working life. Such as the reuse of goods returned to the recycling and distribution complex, as well as the use of damaged goods after repair and restoration. Reusing defective items is preferable to recycling them. Because waste recycling sometimes requires the consumption of energy or raw materials. Therefore, reusing this potential will increase local employment, save resources and energy [15].

Messmann and her colleagues have studied and investigated the repair and reuse of household appliances by poor people (through charity centers) [27]. Some goods (such as durable goods) can be used more than once. In addition, the collected goods can be reused after some minor repairs in second-hand markets, charities or other countries. The aforementioned facts motivated us to focus on a logistics network for the collection and repair of used goods. The main contribution of this research is the design and development of a mathematical optimization model of a reverse logistics network to reuse all kinds of second-hand appliances in small sizes (such as electronic devices and small tablets), medium sizes (such as gas stoves, tables and chairs) and large sizes (such as furniture, refrigerators and freezers). In fact, some charities are stationed in places to donate these items to low-income families. For this purpose, in this research, we intend to present a linear mixed integer two-objective optimization model. The two objective functions are one to optimize the costs of the reverse logistics network (such as transportation and maintenance costs, etc.) and the other to optimize time. In the designed network, the second-hand appliances are transported to the collection centers by their owners or recycling forces. In these centers, products are divided into four main groups, including those in need of repair, recyclables, those that can be sent to charities, and waste materials [29, 31].

2 Research Background

In this section, theoretical foundations and research background are studied in two parts. In the section of theoretical foundations of research, concepts, principles and perspectives related to research terms and variables are presented in detail. In the research background section, are described the conducted researches along with their results.

Humanitarian logistics is a branch of relief logistics that is used in the preparedness and response phases of the crisis management system and is generally expressed as follows: the process of planning, implementation, effective and cost-effective control of the flow and storage of goods and materials. Also, the information related to them, from the point of origin to the point of consumption in order to reduce the pain and suffering of the injured people, that is including a wide range of activities such as preparation, planning, provision of facilities, transportation, storage, routing and clearance of goods from customs [8].

Altiparmak et al. consider humanitarian supply chain management to include the cohesion and cooperation of scattered and disparate groups of experts that can guarantee the main mission of humanitarian aid [2]. People and actors in logistics and relief operations in the humanitarian supply chain are people with different cultures, goals, interests, skills and powers. But the key players are governments, the military, aid agencies, charities, non-profit and voluntary organizations, and the private sector [26].

Dethloff and et al. also proposed a model for creating effective disaster management, which includes five key elements in disaster preparedness, which are human resources, knowledge management, operations and process management, financial resources, and effective results [11]. Also, Dondo and et al. conducted a research in the field of models, solutions and enabling technologies in humanitarian logistics in the response and reconstruction phase [12]. The management cycle of unexpected events is divided into three stages before, during and after the event, each of these stages can be discussed and reviewed separately [17]. The pre-disaster stage includes prevention and preparedness. Prevention means measures that reduce or prevent the risks of accidents [22]. The response phase begins immediately after a disaster and focuses more on saving lives and preventing further damage. At this stage (during a disaster), special attention should be paid to coordination and cooperation between all actors involved in humanitarian relief. The reconstruction stage is the last stage of disaster management and includes helping the affected community to return to its previous state. This stage includes various operations after the disaster, including reconstruction. Also, the purpose of the reconstruction phase is to address the problems of the injured people in a long-term perspective [4, 19].

Anjomshoae et al. in a research, focused on the earthquake event and were able to obtain good practical results. They published their findings in an article [4]. In the above paper, they presented a mathematical method to optimize the reliability of the humanitarian aid supply chain. Reliability and cost management are both important in disaster response. The aim of this research is to increase the reliability and minimize the cost and thus optimize the performance of the humanitarian supply chain. In this research, an integrated reliability optimization model for the relief supply chain is presented. The results of the research showed that the efficiency of crisis operations and the usefulness of humanitarian aid in the crisis situation has increased and the total cost has decreased. They investigated the role of power in supply chain management systems, humanitarian organizations, social and technical systems. Also in their research, they have used the theory of social and technical systems to discover the role of power in the above issues. Their results show that local culture can regulate power relations by identifying the interests of stakeholders and lobbying with donors for the benefit of the supply chain.

Different facility location models are used in the design of logistics networks. Usually, these models are based on mixed integer linear programming. These models include different types of simple models such as locating facilities with unlimited capacity to more complex models such as category models with limited capacity or multi-product models. Also, powerful algorithms based on combinatorial optimization theory have been presented to solve these models. In the field of direct logistics network design, there is a lot of literature, and in this section we mention some related cases. Jayarman and Pirkul in 2001 presented a mixed integer linear programming model for a multi-product and four-tier logistics network [16]. This model is one of the few models that deals with making decisions about suppliers. To solve the problem in this model, an innovative method based on the Lagrange method is presented. In 2005, Melachrinoudis and her colleagues used a multi-objective methodology to redesign the warehouse network structure in order to reduce costs [26]. In this method, the minimum and maximum values are defined for each target, and the minimum weighted sum of the deviation from these values is calculated. In 2006, Amiri presented a mixed integer linear programming model for designing a direct logistics network in a two-tier chain with the aim of cost minimization [3]. What distinguishes Amiri's work from others is the removal of simplifying assumptions such as the unlimited capacity of facilities, the fixed number of facilities, and single-capacity facilities. In 2006, Altiparmak and her colleagues presented a multi-objective nonlinear model for the design of a four-tier supply chain network [2]. Their desired goals are to minimize the cost and maximize the level of responsiveness to customers. The level of responsiveness to the customer means the maximum allowed delivery time and the utilization rate of the distribution centers. Due to the complexity of the problem, it was not possible to design a suitable algorithm to solve it. For this reason, they solved the problem by presenting a genetic algorithm.

In 2007, Lee and Dong designed and presented a model for a logistics network [22]. In the designed model, wastes are divided into two groups of renewable products and scrap products after collection and inspection. Reclaimable products are transported to reclamation centers and there, depending on the quality, either they are remade (repaired) and either with separation operations, their usable parts in Product manufacturing operations are used. Scrap products are transported to recycling and destruction centers and there, depending on the quality, either recycling operations are carried out on them to prepare raw materials, and either if recycling is not possible, safe destruction operations are carried out on them. This network has the ability to support all kinds of industries such as electronic and digital equipment production, vehicle production industries and other similar industries. In a direct network, goods are produced in production centers (such as factories). Then they are transported to distribution centers or warehouses and stored there. In the next step, the goods are transported from warehouses to consumers and delivered. However, in the reverse network, consumer goods are transferred from regions (customers) to collection centers, and from there, after checking the quality, depending on the type of quality, they are transferred to recovery centers or recycling and destruction centers [18]. The reclaimed goods are recirculated in the direct cycle. Thus, the investigated network will be a closed loop logistics network. An important point that should be noted here is that in the investigated network, it is assumed that the recovery of returned products takes place in production centers. Because the products are made in the production centers, their recovery is also possible in the same centers. The flow in direct mode is elastic and dependent on customer demand. And in the reverse mode, it is pressured, based on a percentage of the demand in the direct mode. This means that a fixed percentage of goods flow in the return channel, and on average, a percentage of them are evaluated as salvageable or scrap based on quality. Among the important points in the design of integrated logistics network is the important role of distribution centers in direct flow and collection, inspection and sorting centers in reverse flow. This importance doubles when, in the design stage of the logistics network in an integrated manner, it is possible to establish distribution, inspection and sorting centers in a common place. In the model designed in this treatise, it is possible to create centers in one place, or to build them in non-adjacent places, or some in one place and some in non-adjacent places. Also, in the designed network, in order to match the model with the real state, in addition to the above, several types of capacity have been considered for each facility, if such an issue has not been considered in previous researches.

Reverse logistics is the movement of returned items against the normal direction of goods in the logistics system [25]. In this research, in addition to covering the types of returned items, different ways of recycling, cleaning waste materials, distribution and packaging are also covered. When waste materials cannot be used in the production of other products, they must be disposed of in some way. Also, all production products must be transported and stored efficiently and effectively through the logistics process. Customers are assured that the manufacturer's flexibility to their demand is increased and returned items are accepted more easily. Suppliers are an important member of reverse logistics and closed loop supply chain networks [20]. In classical network models, suppliers are evaluated based on cost and other factors such as on-time delivery are ignored [21]. Suppliers do not allocate discounts to quantity or variety of products, but to the total volume of business. Deep changes and transformations in the world of business and new requirements of production and trade in the current era have provided the basis for the emergence of new attitudes and paradigms that should be taken into consideration by those involved in the field of production and trade. In this regard, a new approach and attitude has been created regarding the issue of logistics under the title of reverse logistics [23, 24]. Based on this, new models for the classification of returned products were presented in 2001 by Dethloff [11] new recovery methods were presented in 2016 by Dondo et al [12]. Dondo and his colleagues designed four types of networks, which are: the direct recovery network of usable products, the renewal or remanufacturing network, the service and repair network, and the recycling network. Jayaraman and colleagues presented a mixed integer linear programming model for reverse logistics network design, which aims to minimize cost [16]. In the field of reverse logistics, there have been other researches, which we mention some of them. Min and et al. in 2006 presented a random mixed integer programming model with the objective of profit maximization for the recycling network [28]. They have developed their model for several modes and considering different scenarios. Also, in 2011, Shi and et al. have presented an advanced multi-product mixed integer programming model by integrating distribution centers with collection centers and recovery centers for closed-loop logistics network design [29]. The goals of their model are cost minimization and customer service time minimization. They proposed a hybrid sparse search method to solve their model. In recent years, researches have been conducted on the design of integrated distribution network (in direct and reverse logistics modes simultaneously). In 2007, Ko and Evans presented a mixed integer nonlinear programming model for integrated logistics network design [19].

3 Modeling of Problem

We have considered types of variables for our model.

- $x \in \mathbb{R}^n$: It represents the vector of decision variables and is subject to the realization of uncertain parameters known as design variables.
- $y \in \mathbb{R}^m$: It represents control decision variables and is susceptible to setting uncertain parameters.

The optimal values of these variables depend on the realization of the uncertain parameters and the optimal values of the design variables.

The optimization model has the following structure:

$$\begin{aligned} \min \quad & Z = C^T x + D^T y, \\ s.t. \quad & Ax = b, \\ & Bx + Cy = e, \\ & x, y \geq 0, \end{aligned}$$

which

$$C = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \quad D = \begin{bmatrix} d_1 \\ \vdots \\ d_m \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix},$$

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mn} \end{bmatrix}, \quad C = \begin{bmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{m1} & \cdots & c_{mm} \end{bmatrix}, \quad e = \begin{bmatrix} e_1 \\ \vdots \\ e_m \end{bmatrix}.$$

The first adverb expresses the structural limitations and its coefficients are constant without any noise and fluctuations. The second adverb expresses the control limits and its coefficients are noisy and fluctuating. The last condition guarantees that the variables are acceptable. $C^T x$ and $D^T y$ are functions that are defined according to the structure of the problem and

$$E^T = (\sum_{j=1}^{n} a_{1j}x_j - b_1, \ldots, \sum_{j=1}^{n} a_{mj}x_j - b_m, \sum_{j=1}^{n} b_{1j}x_j + \sum_{j=1}^{m} c_{1j}y_j - e_1, \ldots, \sum_{j=1}^{n} b_{mj}x_j + \sum_{j=1}^{m} c_{mj}y_j - e_m),$$

is the error vector. which measures the permissible unreasonableness in the control clauses under a scenario. If the scenario is close to optimal, then the optimal solution of the mathematical model is considered robust. If the solution is almost feasible, then we say that the model is in robust state. It is very unlikely that the solution will be both feasible and optimal for all scenarios. The increase in urban population, change in lifestyle, change in diet, as well as increase in the level of well-being and living standards in urban communities have caused a large amount of solid waste in big cities. Currently, managing the process of solid waste in metropolises is one of the most important problems in developing countries. Most of the studies in the literature focus on reverse logistics, for one type of product, in line with the recovery or re-creation process, and not much attention has been paid to the reuse distribution network through charities. In this study, a framework for reusing all kinds of household appliances is proposed to reduce urban solid waste and help low-income families. To optimize the reverse logistics network, a mixed integer linear programming model with uncertainty in the number of products used by consumers is presented. Logistics network designers consider different recycling options such as recycling and repair and centers called charity. In order to show the performance of the presented model, a numerical example is solved by the software to obtain the network structure with optimal cost. The results confirm the applicability of the model by providing a large number of second-hand products that can be transported and reused at an affordable cost. Studies conducted on waste by environmental experts showed that the global standard of per capita waste production per person is 350 grams per day, 70% of which can be recovered. Therefore, the need to create an infrastructure for recovery (recycling or reuse) of used goods is very important. In this research, a reverse logistics network including consumers, local collection centers, regional warehouses, recycling centers, repair centers, and charity centers has been designed for a variety of second-hand appliances under conditions of uncertainty. The flow of products among different facilities are decision variables. In the designed network, second-hand appliances are transferred by consumers or collection service to local collection centers and all of them are sent to the main collection centers. In these centers, products are divided into four main groups, including those in need of repair, recyclable products, products that should be sent to charity centers for reuse, and waste materials (which should be sent to a landfill or waste disposal center). The products in the regional collection centers should be sent to the repair center if they need to be repaired. The number of repair shops and their types are different, including repair shops for electronic devices or repair shops for wooden products. After repair, the products are sent to charity centers. A group of good quality products suitable for reuse are donated directly to charities. Products that can be recycled are taken to recycling centers. While non-recyclable, non-repairable or non-donable products are sent to disposal centers. All products are divided into three groups by size. Small products such as electrical appliances and small tables, medium products such as stoves, tables and chairs, and large products such as furniture and large refrigerators. In this research, we want to examine a structure that includes seven partite with the names of consumers, local collection centers, collection centers, repair centers, charity centers, recycling centers, and disposal centers. We determine the name of the sections and the number of members of each section as follows.

1. m: Consumer, M: Number of consumers
2. k: Local collection center, K: Number of local collection centers
3. j: Collection center, J: Number of collection centers
4. q: Repair center, Q: Number of repair centers
5. e: Charity center, E: Number of charity centers
6. b: Recycling center, B: Number of recycling centers
7. r: Disposal center, R: Number of disposal centers

Now we make a seven-partite graph with the following properties. First, we specify the seven partite of the graph as follows: Partite One: Consumers, Partite two: Local collection centers, Partite three: Collection centers, Partite four: Repair centers, Partite five: Disposal centers, Partite six: Recycling centers, Partite Seven: Charity centers.

1. Members of Partite One (m): Consumers:

m_1	m_2	$\cdots$	m_M

$$d(m_1) = K, \qquad l = 1,2,\ldots,M$$

$$\begin{cases} in-\deg(m_1) = 0, & l = 1,2,\ldots,M \\ out-\deg(m_1) = K, & l = 1,2,\ldots,M \end{cases}$$

2. Members of Partite Two (k): Local collection centers

k_1	k_2	$\cdots$	k_K

$$d(k_l) = M+J, \qquad l = 1,2,\ldots,K$$

$$\begin{cases} in-\deg(k_l) = M, & l = 1,2,\ldots,K \\ out-\deg(k_1) = J, & l = 1,2,\ldots,K \end{cases}$$

3. Members of Partite Three (j): Collection centers

j_1	j_2	$\cdots$	j_J

$$d(j_l) = K+Q+E+B+R, \qquad l = 1,2,\ldots,J$$

$$\begin{cases} in-\deg(j_l) = M, & l = 1,2,\ldots,J \\ out-\deg(j_1) = J, & l = 1,2,\ldots,J \end{cases}$$

4. Members of Partite Four (q): Repair centers

q_1	q_2	$\cdots$	q_Q

$$d(q_l) = J+R+B+E, \qquad l = 1,2,\ldots,Q$$

$$\begin{cases} in-\deg(q_l) = J, & l = 1,2,\ldots,Q \\ out-\deg(q_1) = R+B+E, & l = 1,2,\ldots,Q \end{cases}$$

5. Members of Partite Five (e): Charity centers

e_1	e_2	$\cdots$	e_E

$$d(e_l) = J+Q+B, \qquad l = 1,2,\ldots,E$$

$$\begin{cases} in-\deg(e_l) = J+Q+B, & l = 1,2,\ldots,E \\ out-\deg(e_1) = 0, & l = 1,2,\ldots,E \end{cases}$$

6. Members of Partite Six (b): Recycling centers

b_1	b_2	$\cdots$	b_B

$$d(b_l) = J+Q+E+R, \qquad l = 1,2,\ldots,B$$

$$\begin{cases} in-\deg(b_l) = J+Q, & l = 1,2,\ldots,E \\ out-\deg(b_1) = E+R, & l = 1,2,\ldots,E \end{cases}$$

r_1	r_2	$\cdots$	r_R

7. Members of Partite Seven (r): Disposal centers

$$d(r_l) = J+Q+B, \qquad l=1,2,\ldots,R$$

$$\begin{cases} in-\deg(r_l) = J+Q+B, & l=1,2,\ldots,R \\ out-\deg(r_1) = 0, & l=1,2,\ldots,R \end{cases}$$

Here, we define the adjacency of nodes as follows.

1. The nodes of the graph are as follows. The nodes of Section one: consumers, nodes of section two: local collection centers, nodes of section three: collection centers, nodes of section four: repair centers, nodes of section five: charity centers, nodes of section six: recycling centers and nodes of section seven: disposal centers.

2. There is a directed edge from every node of partite one to all nodes of partite two.

3. There is a directed edge from every node of partite two to all nodes of partite three.

4. There is a directed edge from every node of partite three to all nodes of partite four.

5. There is a directed edge from every node of partite three to all nodes of partite five.

6. There is a directed edge from every node of partite three to all nodes of partite six.

7. There is a directed edge from every node of partite three to all nodes of partite seven.

8. There is a directed edge from every node of partite four to all nodes of partite five.

9. There is a directed edge from every node of partite four to all nodes of partite six.

10. There is a directed edge from every node of partite four to all nodes of partite seven.

11. There is a directed edge from every node of partite six to all nodes of partite five.

12. There is a directed edge from every node of partite six to all nodes of partite seven.

In this way, a graph with $\nu = M+K+J+Q+E+B+R$ nodes and $\varepsilon = MK+KJ+JQ+JE+JB+JR+QR+QE+QB+BE+BR$ edges is created.
Now the question arises whether such a graph can exist? To answer this question, we present the following lemma.

Lemma 1. *Graph with $\nu = M+K+J+Q+E+B+R$ number of nodes and $\varepsilon = MK+KJ+JQ+JE+JB+JR+QR+QE+QB+BE+BR$ number of edges can exist.*

Proof. According to the degree of the nodes, we must have:

$$\begin{aligned} 2\varepsilon &= \sum_{i=1}^{\nu} d(w_i) \\ &= (MK)+(K(M+J))+(J(K+Q+E+B+R))+(Q(J+R+B+E))+(E(J+Q+B))+(B(J+E+Q+R))+(R(J+Q+B)) \\ &= (MK)+(KM+KJ)+(JK+JQ+JE+JB+JR)+(QJ+QR+QB+QE)+(EJ+EQ+EB)+(BJ+BE+BQ+BR)+(RJ+RQ+RB) \\ &= 2(MK+KJ+JQ+JE+JB+JR+QR+QB+QE+EB+BR) = 2\varepsilon \end{aligned}$$

□

Obviously, this graph is connected. Therefore, if we define flow for this connected graph, then is produced a network flow.

In this section, we discuss the modeling of the designed network. For this purpose, we define the following symbols.

$K =$ Number of local collection centers, $J =$ Number of collection centers, $E =$ Number of charity centers, $M =$ Number of consumers, $R =$ Number of disposal centers, $B =$ Number of recycling centers, $Q =$ Number of repair centers, $T =$ Number of commodities, $N =$ Number of vehicles.

According to the above description, the network has ν nodes and ε edges as follows.

$\nu = M+K+J+Q+E+B+R$, $\varepsilon = MK+KJ+JQ+JE+JB+JR+QR+QE+QB+BE+BR$, t: kind of commodity, n: type of vehicle.

Now, after determining the indices, we also define symbols for the parameters.

s_t: The amount of commodity of t used by consumers.

$t_{\max}$: The maximum amount of time that can be allocated to complete all network activities.

$t_{\gamma}^{\max}$ The maximum time required to build the center γ.

c_i^k: The cost of building $i^t h$ local collection center of k.

c_i^j: The cost of building $i^t h$ collection center of j.

c_i^q: The cost of building $i^t h$ repair center of q.

c_i^e: The cost of building $i^t h$ charity collection center of e.

c_i^b: The cost of building $i^t h$ recycling center of b.

c_i^r: The cost of building $i^t h$ disposal center of r.

z_{tk}: The capacity of local collection center of k for commodity of t.

z_{ti}^j: The capacity $i^t h$ of collection center of j for commodity of t.

z_{ti}^q: The capacity $i^t h$ of repair center of q for commodity of t.

z_{ti}^e: The capacity $i^t h$ of charity center of e for commodity of t.

z_{ti}^b: The capacity $i^t h$ of recycling center of b for commodity of t.

z_{ti}^r: The capacity $i^t h$ of disposal center of r for commodity of t.

c_{tk}: The cost of collecting of a unit commodity of t at the local collection center of k.

c_{tj}: The cost of collecting of a unit commodity of t at the collection center of j.

c_{tq}: The cost of repair a unit of commodity of t in the repair center q.

c_{tr}: The cost of disposal a unit of commodity of t in the disposal center r.

c_{tb}: The cost of recycling a unit of commodity of t in the recycling center b.

$e_\tau(\tau = m,k,j,q,e,b,r)$: The total of commodities entering node τ minus the total of commodities leaving node τ. c_{tmjn}: The cost of transporting a unit commodity t from the consumer m to the collection center j by vehicle n.

c_{tkjn}: The cost of transporting a unit commodity t from the consumer m to the collection center j by vehicle n from the local collection center of type k to the collection center of type j by vehicle of n.

c_{tkbn}: The cost of transporting a unit of commodity of t from a local collection center of k to a recycling location of b by a vehicle of n.

c_{tkqn}: The cost of transporting of a unit of commodity of t from the local collection center of k to the repair center of q by vehicle of n.

c_{tken}: The cost of transporting a unit of commodity t from the local collection center of k to the charity center of e by vehicle of n.

c_{tkrn}: The cost of transporting a unit commodity t from the local collection center of k from the disposal center of by vehicle of n.

c_{tqen}: The cost of transporting of a unit commodity t from repair center of q to charity center of e by vehicle of n.

w_{tmkn}: The cost of collection (in the local collection center of k) of a unit of commodity of t sent from the center of m to the center of k by vehicle of n.

w_{tkjn}: The cost of collection (in the collection center of j) of a unit of commodity of t sent from the center of k to the center of j by vehicle of n.

w_{tjqn}: The cost of repair (in the repair center of q) of a unit of commodity of t sent from the center of j to the center of q by vehicle of n.

w_{tjen}: The cost of assignment (in the charity center of e) of a unit of commodity of t sent from the center of j to the center of e by vehicle of n.

w_{tjbn}: The cost of recycling (in the recycling center of b) of a unit of commodity of t sent from the center of j to the center of b by vehicle of n.

w_{tjrn}: The cost of disposal (in the disposal center of r) of a unit of commodity of t sent from the center of j to the center of r by vehicle of

n.

w_{tqen}: The cost of assignment (in the charity center of e) of a unit of commodity of t sent from the center of q to the center of e by vehicle of n.

w_{tqbn}: The cost of recycling (in the recycling center of b) of a unit of commodity of t sent from the center of q to the center of b by vehicle of n.

w_{tqrn}: The cost of disposal (in the disposal center of r) of a unit of commodity of t sent from the center of q to the center of r by vehicle of n.

w_{tbrn}: The cost of disposal (in the disposal center of r) of a unit of commodity of t sent from the center of b to the center of r by vehicle of n.

$w_t ben$: The cost of assignment (in the charity center of e) of a unit of commodity of t sent from the center of b to the center of e by vehicle of n.

d_1: The percentage of commodities transferred from collection centers to repair centers.

d_2: The percentage of commodities transferred from collection centers to charity centers.

d_3: The percentage of commodities transferred from collection centers to recycling centers.

d_4: The percentage of commodities transferred from collection centers to disposal centers.

z_n: Vehicle capacity n, v_t: Commodity volume of t.

Now,after determining the parameters,we determine the decision variables.

x_{tkjn}: Amount of transportation of commodity of t from local collection center of k to collection center of j by vehicle of n.

x_{tjqn}: Amount of transportation of commodity of t from collection center of j to repair center of q by vehicle of n.

x_{tjen}: Amount of transportation of commodity of t from collection center of j to charity center of e by vehicle of n.

x_{tjbn}: Amount of transport of commodity of t from collection center of j to recycling center of b by vehicle of n.

x_{tjrn}: Amount of transportation of commodity of t from collection center of j to disposal center of r by vehicle of n.

x_{tqbn}: Amount of transportation of commodity of t from repair center of q to recycling center of b by vehicle of n.

x_{tqen}: Amount of transportation of commodity of t from repair center of q to charity center of e by vehicle of n.

x_{tqrn}: Amount of transportation of commodity of t from repair center of q to disposal center of r by vehicle of n.

x_{tbrn}: Amount of transportation of commodity of t from recycling center of b to disposal center of r by vehicle of n.

x_{tben}: Amount of transportation of commodity of t from recycling center of b to charity center of e by vehicle of n.

We show $i^t h$ of the center of j with u_i^j.

y_i^j: The defining variable of the construction of the center u_i^j.

$$y_i^j = \begin{cases} 1, & \text{If center } \mu_i^j \text{ is built,} \\ 0, & \text{Otherwise.} \end{cases}$$

t_i^j: Time needed to build the center ^j_i.

t_{tjen}: The time required to transfer (one transfer) of commodity of t from the collection center of j to the charity center of e by vehicle of n.

π_{tjen}: The number of transfers that vehicle of n needs to carry commodity of t from center of j to center of e.

t_{tqen}: The time required to transfer (one transfer) of commodity of t from the repair center of q to the charity center of e by vehicle of n.

π_{tqen}: The number of transfers that vehicle of n needs to carry commodity of t from center of q to center of e.

t_{tkjn}: The time required to transfer (one transfer) of commodity of t from local collection center of k to collection center of j by vehicle of n.

π_{tkjn}: The number of transfers that vehicle of n needs to carry commodity of t from center of k to center of j.

t_{tjqn}: The time required to transfer (one transfer) of commodity of t from collection center of j to repair center of q by vehicle of n.

π_{tjqn}: The number of transfers that vehicle of n needs to carry commodity of t from center of j to center of q.

t_{tjrn}: The time required to transfer (one transfer) of commodity of t from collection center of j to disposal center of r by vehicle of n.

π_{tjrn}: The number of transfers that vehicle of n needs to carry commodity of t from center of j to center of r.

t_{tjbn}: The time required to transfer (one transfer) of commodity of t from collection center of j to recycling center of b by vehicle of n.

π_{tjbn}: The number of transfers that vehicle of n needs to carry commodity of t from center of j to center of b.

t_{tben}: The time required to transfer (one transfer) of commodity of t from recycling center of b to charity center of e by vehicle of n.

π_{tben}: The number of transfers that vehicle of n needs to carry commodity of t from center of b to center of e.
t_{tbrn}: The time required to transfer (one transfer) of commodity of t from recycling center of b to disposal center of r by vehicle of n.
π_{tbrn}: The number of transfers that vehicle of n needs to carry commodity of t from center of b to center of r.
t_{tqrn}: The time required to transfer (one transfer) of commodity of n from repair center of q to disposal center of r by vehicle of n.
π_{tqrn}: The number of transfers that vehicle of n needs to carry commodity of t from center of q to center of r.
t_{tqbn}: The time required to transfer (one transfer) of commodity of t from repair center of q to recycling center of b by vehicle of n.
π_{tqbn}: The number of transfers that vehicle of n needs to carry commodity of t from center of q to center of b.
t'_{tmkn}: The time required to collect (at the local collection center of k) a unit of commodity of t transferred from the center of m to the center of k by the vehicle of n.
t'_{tkjn}: The time required to collect (at the collection center of j) a unit of commodity of t transferred from the center of k to the center of j by the vehicle of n.
t'_{tjqn}: The time required to repair (at the repair center of q) a unit of commodity of t transferred from the center of j to the center of q by the vehicle of n.
t'_{tjen}: The time required to assignment (at the charity center of e) a unit of commodity of t transferred from the center of j to the center of e by the vehicle of n.
t'_{tjbn}: The time required to recycling (at the recycling center of b) a unit of commodity of t transferred from the center of j to the center of b by the vehicle of n.
t'_{tjrn}: The time required to disposing (at the disposing center of r) a unit of commodity of t transferred from the center of j to the center of r by the vehicle of n.
t'_{tqbn}: The time required to recycling (at the recycling center of b) a unit of commodity of t transferred from the center of q to the center of b by the vehicle of n.
t'_{tqen}: The time required to assignment (at the charity center of e) a unit of commodity of t transferred from the center of q to the center of e by the vehicle of n.
t'_{tqrn}: The time required to disposing (at the disposing center of r) a unit of commodity of t transferred from the center of q to the center of r by the vehicle of n.
t'_{tben}: The time required to assignment (at the charity center of e) a unit of commodity of t transferred from the center of b to the center of e by the vehicle of n.
t'_{tbrn}: The time required to disposing (at the disposing center of r) a unit of commodity of t transferred from the center of b to the center of r by the vehicle of n.
Now, after determining the indices, parameters and decision variables, we present the mathematical model.

Now we calculate the costs of transporting the commodities.

$$
\begin{aligned}
&F_1=\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N}c_{tkjn}x_{tkjn}, \quad F_2=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N}c_{tjqn}x_{tjqn},\\
&F_3=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N}c_{tjen}x_{tjen}, \quad F_4=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N}c_{tjbn}x_{tjbn},\\
&F_5=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N}c_{tjrn}x_{tjrn}, \quad F_6=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N}c_{tqbn}x_{tqbn},\\
&F_7=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N}c_{tqen}x_{tqen}, \quad F_8=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N}c_{tqrn}x_{tqrn},\\
&F_9=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N}c_{tbrn}x_{tbrn}, \quad F_{10}=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N}c_{tben}x_{tben},\\
&C_1=\sum_{i=1}^{10}F_i
\end{aligned}
$$

Now we calculate the costs of building the centers.

$$
\begin{aligned}
&G_1=\sum_{i=1}^{K}c_i^k y_i^k, \quad G_2=\sum_{i=1}^{J}c_i^j y_i^j, \quad G_3=\sum_{i=1}^{Q}c_i^q y_i^q, \quad G_4=\sum_{i=1}^{E}c_i^e y_i^e,\\
&G_5=\sum_{i=1}^{B}c_i^b y_i^b, \quad G_6=\sum_{i=1}^{R}c_i^r y_i^r, \quad C_2=\sum_{\sigma=1}^{6}G_\sigma
\end{aligned}
$$

Now we calculate the costs of collecting commodities in collection centers, the costs of repairing commodities in repair centers, the costs of assignment commodities to individuals in charity centers, the costs of recycling commodities in recycling centers and the costs of disposing of commodities in disposal centers.

$$W_1=\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}w_{tmkn}x_{tmkn},\quad W_2=\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N}w_{tkjn}x_{tkjn},$$
$$W_3=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N}w_{tjqn}x_{tjqn},\quad W_4=\sum_{t=1}^{N}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N}w_{tjen}x_{tjen},$$
$$W_5=\sum_{t=1}^{N}\sum_{j=1}^{N}\sum_{b=1}^{B}\sum_{n=1}^{N}w_{tjbn}x_{tjbn},\quad W_6=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N}w_{tjrn}x_{tjrn},$$
$$W_7=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N}w_{tqen}x_{tqen},\quad W_8=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N}w_{tqbn}x_{tqbn},$$
$$W_9=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N}w_{tqrn}x_{tqrn},\quad W_{10}=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N}w_{tbrn}x_{tbrn},$$
$$W_{11}=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N}w_{tben}x_{tben},\quad C_3=\sum_{\theta=1}^{11}W_\theta$$

Now we define the functions that calculate the required time to transfer the commodities and also the required number of transfers of each vehicle in the network.

$$T_1=\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N}t_{tkjn}\pi_{tkjn},\quad T_2=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N}t_{tjqn}\pi_{tjqn},$$
$$T_3=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N}t_{tjen}\pi_{tjen},\quad T_4=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N}t_{tjbn}\pi_{tjbn},$$
$$T_5=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N}t_{tjrn}\pi_{tjrn},\quad T_6=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N}t_{tqbn}\pi_{tqbn},$$
$$T_7=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N}t_{tqen}\pi_{tqen},\quad T_8=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N}t_{tqrn}\pi_{tqrn},$$
$$T_9=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N}t_{tbrn}\pi_{tbrn},\quad T_{10}=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N}t_{tben}\pi_{tben},$$
$$\check{T}_1=\sum_{\beta=1}^{10}T_\beta$$

Now we calculate the time required to build the centers.

$$\check{T}_2=\sum_{\gamma\in\Omega}\sum_{i=1}^{|\gamma|}t_i^{\gamma},\qquad |\gamma|:\text{Numberofcenters}\gamma$$

Now we calculate the times of collecting commodities in collection centers, the times of repairing commodities in repair centers, the times of assignment commodities to individuals in charity centers, the times of recycling commodities in recycling centers and the times of disposing of commodities in disposal centers

$$T_1'=\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N}t'_{tkjn}x_{tkjn},\quad T_2'=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N}t'_{tjqn}x_{tjqn},$$
$$T_3'=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N}t'_{tjen}x_{tjen},\quad T_4'=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N}t'_{tjbn}x_{tjbn},$$
$$T_5'=\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N}t'_{tjrn}x_{tjrn},\quad T_6'=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N}t'_{tqbn}x_{tqbn},$$
$$T_7'=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N}t'_{tqen}x_{tqen},\quad T_8'=\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N}t'_{tqrn}x_{tqrn},$$
$$T_9'=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N}t'_{tbrn}x_{tbrn},\quad T_{10}'=\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N}t'_{tben}x_{tben},$$
$$\check{T}_3=\sum_{\delta=1}^{10}T_\delta'$$

The above functions calculate the total costs of local and main collection centers, charity centers, repair centers, recycling centers, disposal centers, as well as the costs of sending commodities between different centers of the network.

Now we specify the restrictions of the problem.

$$\sum_{m=1}^{M}x_{tmkn}=\sum_{k=1}^{K}x_{tkjn},\qquad t=1,2,\ldots,T,\ j=1,2,\ldots,J,\ n=1,2,\ldots,N \tag{1}$$

Restrictions (1) states that the total number commodity of t sent from consumers to collection center of k by vehicle of n must be equal to the total number of commodity of t sent from local collection centers to collection center of j by vehicle of n.

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}x_{tmkn}=s_t,\qquad t=1,2,\ldots,T \tag{2}$$

Restrictions (2) states that the total number commodity of t sent from consumers to local collection centers by vehicles must be equal to the number of consumer commodity of t.

$$d_1\sum_{j=1}^{J}x_{tkjn}\leq\sum_{j=1}^{J}x_{tjqn},\ n=1,2,\ldots,N,\ t=1,2,\ldots,T,\ k=1,2,\ldots,K,\ q=1,2,\ldots,Q \tag{3}$$

Restrictions (3) state that the total of commodity of t sent from the local collection center of k to the collection centers by the vehicle of n must be less than or equal to the total of the commodity of t sent from the collection centers to the repair center of q by The vehicle of n.

$$d_2 \sum_{j=1}^{J} x_{tkjn} \leq \sum_{j=1}^{J} x_{tjen},\ n = 1,2,\dots,N,\ t = 1,2,\dots,T,\ k = 1,2,\dots,K,\ e = 1,2,\dots,E \tag{4}$$

Restrictions (4) state that the total of commodity of t sent from the local collection center of k to the collection centers by the vehicle of n must be less than or equal to the total of the commodity of t sent from the collection centers to the charity center of e by the vehicle of n.

$$d_3 \sum_{j=1}^{J} x_{tkjn} \leq \sum_{j=1}^{J} x_{tjbn},\ n = 1,2,\dots,N,\ t = 1,2,\dots,T,\ k = 1,2,\dots,K,\ b = 1,2,\dots,B \tag{5}$$

Restrictions (5) state that the total of commodity of t sent from local collection center of k to collection centers by vehicle of n must be less than or equal to the total of commodity of t sent from collection centers to recycling center of b by vehicle of n.

$$d_4 \sum_{j=1}^{J} x_{tkjn} \leq \sum_{j=1}^{J} x_{tjrn},\ n = 1,2,\dots,N,\ t = 1,2,\dots,T,\ k = 1,2,\dots,K,\ r = 1,2,\dots,R \tag{6}$$

Restrictions (6) state that the total of commodity of t sent from the local collection center of k to the collection centers by the vehicle of n must be less than or equal to the total of the commodity of t sent from the collection centers to the disposal centers of r by vehicle of n.

$$\sum_{j=1}^{J} x_{tjqn} \leq \sum_{q=1}^{Q} x_{tqen},\ t = 1,2,\dots,T,\ n = 1,2,\dots,N,\ q = 1,2,\dots,Q,\ e = 1,2,\dots,E \tag{7}$$

Restrictions (7) state that the total amount of commodity of t sent from collection centers to repair center of q by vehicle of n must be less than or equal to the total amount of g commodity of t sent from repair centers to charity centers of e by means of transportation of n.

$$\sum_{j=1}^{J} x_{tjbn} \leq \sum_{b=1}^{B} x_{tben},\ t = 1,2,\dots,T,\ n = 1,2,\dots,N,\ b = 1,2,\dots,B,\ e = 1,2,\dots,E \tag{8}$$

Restrictions (8) state that the total amount of commodity of t sent from collection centers to recycling center of b by vehicle of n must be less than or equal to the total amount of commodity of t sent from recycling centers to charity center of e by means of transportation of n.

$$\sum_{t=1}^{T} \sum_{k=1}^{K} \sum_{n=1}^{N} x_{tkin} \leq y_i^j \sum_{t=1}^{T} z_{ti}^j, \qquad j = 1,2,\dots,J \tag{9}$$

Restrictions (9) state that the total amount of commodities sent from the local collection center of k to the collection centers by vehicles must be smaller than or equal to the capacity of the local collection center of k.

$$\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{n=1}^{N} x_{tjin} \leq \sum_{i=1}^{E} y_i^e \sum_{t=1}^{T} z_{ti}^e, \qquad i = 1,2,\dots,E \tag{10}$$

Restrictions (10) state that the total amount of commodities sent from collection centers to charity center of e by vehicles must be smaller than or equal to the capacity of charity center of e.

$$\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{n=1}^{N} x_{tjin} \leq \sum_{i=1}^{Q} y_i^q \sum_{t=1}^{T} z_{ti}^q, \qquad i = 1,2,\dots,Q \tag{11}$$

Restrictions (11) state that the total amount of commodities sent from collection centers to repair center of q by vehicles must be smaller than or equal to the capacity of repair center of q.

$$\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{n=1}^{N} x_{tjin} \leq y_i^b \sum_{t=1}^{T} z_{ti}^b, \qquad i = 1,2,\dots,B \tag{12}$$

Restrictions (12) state that the total amount of commodities sent from collection centers to recycling center of b by vehicles must be smaller than or equal to the capacity of recycling center of b.

$$\sum_{t=1}^{T} \sum_{j=1}^{J} \sum_{n=1}^{N} x_{tjin} \leq y_i^r \sum_{t=1}^{T} z_{ti}^r, \qquad i = 1,2,\dots,R \tag{13}$$

Restrictions (13) state that the total amount of commodities sent from collection centers to disposal center of r by vehicles must be smaller than or equal to the capacity of disposal center of r.

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N} x_{tmin} \leq y_i^k \sum_{t=1}^{T} z_{ti}^k, \qquad i = 1,2,\ldots,K \tag{14}$$

Restrictions (14) state that the total amount of commodities sent from consumers to local collection center of k by vehicles must be smaller than or equal to the capacity of local collection center of k.

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N} v_t x_{tmkn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{15}$$

Restrictions (15) state that the total amount of commodities sent from consumers to collection centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J} v_t x_{tkjn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{16}$$

Restrictions (16) state that the total amount of commodities sent from local collection centers to collection centers by vehicle of n must be smaller than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B} v_t x_{tjbn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{17}$$

Restrictions (17) state that the total amount of commodities sent from local collection centers to recycling centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q} v_t x_{tjqn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{18}$$

Restrictions (18) state that the total amount of commodities sent from local collection centers to repair centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E} v_t x_{tjen} \leq z_n, \qquad n = 1,2,\ldots,N \tag{19}$$

Restrictions (19) state that the total amount of commodities sent from local collection centers to charity centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R} v_t x_{tjrn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{20}$$

Restrictions (20) state that the total amount of commodities sent from local collection centers to disposal centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{Q} v_t x_{tqen} \leq z_n, \qquad n = 1,2,\ldots,N \tag{21}$$

Restrictions (21) state that the total amount of commodities sent from repair centers to charity centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B} v_t x_{tqbn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{22}$$

Restrictions (22) state that the total amount of commodities sent from repair centers to recycling centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R} v_t x_{tqrn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{23}$$

Restrictions (23) state that the total amount of commodities sent from repair centers to disposal centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R} v_t x_{tbrn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{24}$$

Restrictions (24) state that the total amount of commodities sent from recycling centers to disposal centers by vehicle of n must be less than or equal to the capacity of vehicle of n.

$$\begin{aligned}
&\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} t_{tmkn}\pi_{tmkn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tjrn}\pi_{tjrn} + \\
&\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} t_{tkjn}\pi_{tkjn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} t_{tjqn}\pi_{tjqn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tjen}\pi_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tjbn}\pi_{tjbn} + \\
&\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tqbn}\pi_{tqbn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tqen}\pi_{tqen} + \\
&\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tbrn}\pi_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tben}\pi_{tben} + \\
&\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} t'_{tkjn}x_{tkjn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} t'_{tjqn}x_{tjqn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tjen}x_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} t'_{tjbn}\pi_{tjbn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tjrn}x_{tjrn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} t'_{tqbn}x_{tqbn} + \\
&\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tqen}x_{tqen} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tqrn}x_{tqrn} + \\
&\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tbrn}x_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tben}x_{tben} + \\
&\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tqrn}\pi_{tqrn} + \sum_{\gamma\in\Omega}\sum_{i=1}^{|\gamma|} t_i^{\gamma} \leq t_{\max}
\end{aligned} \tag{25}$$

Restriction (25) guarantees that the time required to perform all network activities must be less than or equal to the maximum time allocated to perform all network activities.

Now we present the capacity balance constraints of the nodes.

Node k:

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N} x_{tmkn} - \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tkjn} = e_k, \qquad k = 1,2,\ldots,K \tag{26}$$

Node j:

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{n=1}^{N} x_{tkjn} - (\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{n=1}^{N} x_{tjqn} + \sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{n=1}^{N} x_{tjen} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N} x_{tjbn} + \sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{n=1}^{N} x_{tjrn}) = e_j, \qquad j = 1,2,\ldots,J \tag{27}$$

Node q:

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tjqn} + (\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N} x_{tqbn} + \sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{n=1}^{N} x_{tqen} + \sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{n=1}^{N} x_{tqrn}) = e_q, \qquad q = 1,2,\ldots,Q \tag{28}$$

Node e:

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{n=1}^{N} x_{tqen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tjen} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N} x_{tben} = e_e, \qquad e = 1,2,\ldots,E \tag{29}$$

Node b:

$$(\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N}x_{tjbn}+\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{n=1}^{N}x_{tqbn})-(\sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{n=1}^{N}x_{tben}+\sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{n=1}^{N}x_{tbrn})=e_b, \qquad b=1,2,\dots,B \tag{30}$$

Node r:

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N}x_{tjrn}+\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{n=1}^{N}x_{tqrn}+\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N}x_{tbrn}=e_r, \qquad r=1,2,\dots,R \tag{31}$$

Node m:

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{n=1}^{N}x_{tmkn}=-e_m, \qquad m=1,2,\dots,M \tag{32}$$

Now we present the mathematical model of the problem as follows.

$$\min(C_1,C_2,C_3,\check{T}_1,\check{T}_2,\check{T}_3)$$

$$s.t.$$

$$\sum_{m=1}^{M}x_{tmkn}=\sum_{k=1}^{K}x_{tkjn}, \qquad t=1,2,\dots,T,\ j=1,2,\dots,J,\ n=1,2,\dots,N \tag{33}$$

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}x_{tmkn}=s_t, \qquad t=1,2,\dots,T \tag{34}$$

$$d_1\sum_{j=1}^{J}x_{tkjn}\le\sum_{j=1}^{J}x_{tjqn}, \qquad n=1,2,\dots,N\ t=1,2,\dots,T,\ k=1,2,\dots,K,\ q=1,2,\dots,Q \tag{35}$$

$$d_2\sum_{j=1}^{J}x_{tkjn}\le\sum_{j=1}^{J}x_{tjen}, \qquad n=1,2,\dots,N\ t=1,2,\dots,T,\ k=1,2,\dots,K,\ e=1,2,\dots,E \tag{36}$$

$$d_3\sum_{j=1}^{J}x_{tkjn}\le\sum_{j=1}^{J}x_{tjbn}, \qquad n=1,2,\dots,N\ t=1,2,\dots,T,\ k=1,2,\dots,K,\ b=1,2,\dots,B \tag{37}$$

$$d_4\sum_{j=1}^{J}x_{tkjn}\le\sum_{j=1}^{J}x_{tjrn}, \qquad n=1,2,\dots,N\ t=1,2,\dots,T,\ k=1,2,\dots,K,\ r=1,2,\dots,R \tag{38}$$

$$\sum_{j=1}^{J}x_{tjqn}\le\sum_{q=1}^{Q}x_{tqen}, \qquad t=1,2,\dots,T,\ n=1,2,\dots,N,\ q=1,2,\dots,Q,\ e=1,2,\dots,E \tag{39}$$

$$\sum_{j=1}^{J}x_{tjbn}\le\sum_{b=1}^{B}x_{tben}, \qquad t=1,2,\dots,T,\ n=1,2,\dots,N,\ b=1,2,\dots,B,\ e=1,2,\dots,E \tag{40}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{n=1}^{N}x_{tkin}\le y_i^j\sum_{t=1}^{T}z_{ti}^j, \qquad i=1,2,\dots,J \tag{41}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N}x_{tjin}\le\sum_{i=1}^{E}y_i^e\sum_{t=1}^{T}z_{ti}^e, \qquad i=1,2,\dots,E \tag{42}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N}x_{tjin}\le\sum_{i=1}^{Q}y_i^q\sum_{t=1}^{T}z_{ti}^q, \qquad i=1,2,\dots,Q \tag{43}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N}x_{tjin}\le y_i^b\sum_{t=1}^{T}z_{ti}^b, \qquad i=1,2,\dots,B \tag{44}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N}x_{tjin}\le y_i^r\sum_{t=1}^{T}z_{ti}^r, \qquad i=1,2,\dots,R \tag{45}$$

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N}x_{tmin}\le y_i^k\sum_{t=1}^{T}z_{ti}^k, \qquad i=1,2,\dots,K \tag{46}$$

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}v_t x_{tmkn}\le z_n, \qquad n=1,2,\dots,N \tag{47}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J} v_t x_{tkjn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{48}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B} v_t x_{tjbn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{49}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q} v_t x_{tjqn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{50}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E} v_t x_{tjen} \leq z_n, \qquad n = 1,2,\ldots,N \tag{51}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R} v_t x_{tjrn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{52}$$

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E} v_t x_{tqen} \leq z_n, \qquad n = 1,2,\ldots,N \tag{53}$$

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B} v_t x_{tqbn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{54}$$

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R} v_t x_{tqrn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{55}$$

$$\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R} v_t x_{tbrn} \leq z_n, \qquad n = 1,2,\ldots,N \tag{56}$$

$$\begin{gathered}
\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} t_{tmkn}\pi_{tmkn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tjrn}\pi_{tjrn} + \\
\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} t_{tkjn}\pi_{tkjn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} t_{tjqn}\pi_{tjqn} + \\
\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tjen}\pi_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tjbn}\pi_{tjbn} + \\
\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tqbn}\pi_{tqbn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tqen}\pi_{tqen} + \\
\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tbrn}\pi_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tben}\pi_{tben} + \\
\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} t'_{tkin}x_{tkin} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} t'_{tjqn}x_{tjqn} + \\
\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tjen}x_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} t'_{tjbn}x_{tjbn} + \\
\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tjrn}x_{tjrn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} t'_{tqbn}x_{tqbn} + \\
\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tqen}x_{tqen} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tqrn}x_{tqrn} + \\
\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tbrn}x_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tben}x_{tben} +
\end{gathered}$$

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tbrn}\pi_{tqrn} + \sum_{\gamma\in\Omega}\sum_{i=1}^{|\gamma|} t_i^{\gamma} \leq t_{\max} \tag{57}$$

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{n=1}^{N} x_{tmkn} - \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tkjn} = e_k, \qquad k=1,2,\ldots,K \tag{58}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{N}\sum_{n=1}^{N} x_{tkjn} - (\sum_{t=1}^{T}\sum_{q=1}^{N}\sum_{n=1}^{N} x_{tjqn} + \sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{n=1}^{N} x_{tjen} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N} x_{tjbn} + \sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{n=1}^{N} x_{tjrn}) = e_j, \ j=1,2,\ldots,J \tag{59}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tjqn} - (\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N} x_{tqbn} + \sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{n=1}^{N} x_{tqen} + \sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{n=1}^{N} x_{tqrn}) = e_q, \ q=1,2,\ldots,Q \tag{60}$$

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{n=1}^{N} x_{tqen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tjen} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N} x_{tben} = e_e, \ e=1,2,\ldots,E \tag{61}$$

$$(\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tjbn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{n=1}^{N} x_{tqbn}) - (\sum_{t=1}^{T}\sum_{e=1}^{E}\sum_{n=1}^{N} x_{tben} + \sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{n=1}^{N} x_{tbrn}) = e_b, \ b=1,2,\ldots,B \tag{62}$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{n=1}^{N} x_{tjrn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{n=1}^{N} x_{tqrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{n=1}^{N} x_{tbrn} = e_r, \ r=1,2,\ldots,R \tag{63}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{n=1}^{N} x_{tmkn} = -e_m, \qquad =1,2,\ldots,M \tag{64}$$

$$0 \le x_{tkjn} \le u_{tkjn}, \qquad 0 \le x_{tj\alpha n} \le u_{tj\alpha n}, \ \alpha = q,e,b,r, \tag{65}$$

$$0 \le x_{tq\theta n} \le u_{tq\theta n}, \qquad \theta = b,e,r, \qquad 0 \le x_{tb\gamma n} \le u_{tb\gamma n}, \ \gamma = r,e \tag{66}$$

$$0 \le t_i^{\gamma} \le t_{\gamma}^{\max}, \qquad \gamma \in \Omega, \ i=1,2,\ldots,|\gamma|, \qquad 0 \le t_{\alpha\beta n} \le t_{\gamma}^{\max}, \tag{67}$$

$$0 \le t'_{abcd} \le t_{\gamma}^{\max}, \qquad \pi_{abcd} \ge 0, \qquad \forall, a,b,c,d \tag{68}$$

$$y_i^{\gamma} \in \{0,1\}, \qquad \gamma \in \Omega, \qquad i=1,2,\ldots,|\gamma| \tag{69}$$

The obtained model is a multi-objective programming problem. We will solve this problem in the next chapters.

4 Solving the Problem of Recycling in the Reverse Logistics Network

In the previous section, we modeled the problem of recycling commodities in a kind of reverse logistics network. The obtained model is a nonlinear multi-objective programming model with a number of uncertain parameters. For this reason, we will first present the general form of multi-objective programming problems and then examine a number of methods to solve them.

4.1 Multi-Objective Optimization

A multi-objective optimization problem can be expressed as follows:

$$\begin{array}{lll} \min & (H_1(X), H_2(X), \ldots, H_k(X)) & \\ s.t. & G_j(X) \le 0, & j=1,2,\ldots,m \\ that & X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} & \end{array}$$

k represents the number of objective functions to be minimized. Objective functions and constraint functions may be linear or non-linear.

On the other hand, the problem has a relatively large number of variables and adverbs. For these reasons, it is very difficult and impossible to solve it with accurate analytical methods. Therefore, we use meta-heuristic methods to solve it. For this reason, we first present the definitions, principles, and general structure of meta-heuristic methods. The multi-objective optimization problem is also known as a vector optimization problem. Now we will describe a methods for solving the multi-objective optimization problem [5].

4.1.1 Desirability Function Method

In this method, depending on the importance of H_i $(i=1,2,\ldots,k)$ compared to other functions, we define a desirability function $D_i(H_i)$ $(i=1,2,\ldots,k)$ for each of the objective functions. For example, desirability functions can be defined as follows.

$$D=\sum_{i=1}^{k} D_i(H_i)$$

Then we obtain the answer vector X^* by maximizing (minimizing) $D(-D)$ under the conditions of adverbs $G_j(X)\leq 0, j=1,2,\ldots,m$. A conventional rule for desirability functions is to assign a weight (W_i) to each objective function.

$$D=-\sum_{i=1}^{k} W_i H_i(X)$$

When the importance of the objective functions is the same for us, we can set all the weights equal to 1.

Our designed problem in the previous chapter can be solved by all the above methods. In this research, we solve it using the desirability function method and leave the problem solving to other methods as future work. If for simplicity, we consider the coefficient of desirability of the objective functions to be equal to one, then the problem model will be as follows:

$$\begin{aligned}
&\min\ Z=\sum_{j=1}^{3} C_j+\sum_{j=1}^{3} \check{T}_j\\
&s.t.\\
&\text{Restrictions (33) to (64)}\\
&0\leq x_{tkjn}\leq u_{tkjn}, \qquad 0\leq x_{tj\alpha n}\leq u_{tj\alpha n}, \qquad \alpha=q,e,b,r,\\
&0\leq x_{tq\theta n}\leq u_{tq\theta n}, \qquad \theta=b,e,r, \qquad 0\leq x_{tb\gamma n}\leq u_{tb\gamma n},\ \gamma=r,e\\
&0\leq t_\gamma\leq t_\gamma^{\max}, \qquad \gamma\in\Omega
\end{aligned}$$

Before solving this problem, we first calculate its size.

F_1 = The number of variables of $x_{t\alpha\beta n}(\text{In}C_1)$ = TKKN+TJQN+TJEN+TJBN+TJRN+TQBN+TQEN+TQRN+TBEN+TBRN

F_2 = The number of variables of $y_\gamma(\text{In}C_2)$ =K+J+Q+E+B+R

F_3 = The number of variables of $x_{t\alpha\beta n}(\text{In}C_3)$ = TMKN+TKKN+TJQN+TJEN+TJBN+TJRN+TQBN+TQEN+TQRN+TBEN+TBRN

F_4 = The number of variables of $(\text{In}\check{T}_1)$ = 2 (TMKN+TKKN+TJQN+TJEN+TJBN+TJRN+TQBN+TQEN+TQRN+TBEN+TBRN)

F_5 = The number of variables of $(\text{In}\check{T}_2)$ = K+J+Q+E+B+R

F_6 = The number of variables of $(\text{In}\check{T}_3)$ = 2 (TMKN+TKKN+TJQN+TJEN+TJBN+TJRN+TQBN+TQEN+TQRN+TBEN+TBRN)

F = The total number of variables $\sum_{i=1}^{6} F_i$.

R = Number of c Restrictions = $\sum_{j=1}^{32}$ Number of c Restrictions (j) = TJN+T+TKQN+TKEN+TKBN+TKRN+TQEN+TBEN+J+E+Q+B+R +K+10N+1+K+J+Q+E+B+R+M.

If the number of nodes and edges are few in a reverse recycling network, then the model is small and therefore the use of algorithmic and software methods is not very economical. But in the case that the number of centers is large, the number of edges is naturally also increased, and therefore according to the above calculations (calculations of the number of nodes and edges), the model is large and complex, and it is very difficult to solve it by analytical methods. For this reason, we solve the designed model by one of the meta-heuristic methods.

5 Metaheuristic Algorithms

Meta-heuristic (meta-evolutionary) algorithms are a type of imprecise (not necessarily precise) algorithms that are used to find answers (solution) to problems. Optimization methods and algorithms (from a point of view) are divided into two categories: exact and approximate algorithms. Exact algorithms are able to find the optimal solution accurately, but they are not effective in hard optimization problems and their solving time increases exponentially in these problems. Approximate algorithms are able to find good solutions (close to the optimal solution) in a short (suitable) time for hard optimization problems. Approximate algorithms are divided into three categories: heuristic, meta-heuristic and super-heuristic. The two main problems of heuristic algorithms are their focus on local optimal points and their lack of use in different problems. Meta-heuristic algorithms are presented to reduce (eliminate) the shortcomings of heuristic algorithms. Actually, meta-heuristic algorithms are one of the good approximate algorithms for hard optimization problems. which have exit solutions from local optimal points and can be used in a wide range of difficult (hard) problems. Various types of this type of algorithm have been developed in recent decades. One of the most famous and widely used of them is the genetic algorithm (GA) [1,6,9,10,30].

5.1 Designed Genetic Algorithm

Before presenting the genetic algorithm that we have designed for our problem, it is necessary to make a preliminary statement. First, we form the chromosomal strings corresponding to the decision variables.

1. For $x_{t\alpha\beta n}$
   $\acute{R} =$ The number of $x_{t\alpha\beta n}$

$x_{t\alpha\beta n}:$	$x^1_{t\alpha\beta n}$	$x^2_{t\alpha\beta n}$	$\cdots$	$x^{\acute{R}}_{t\alpha\beta n}$

   $\check{N}_1 =$ The number of arrays of string related to $x_{t\alpha\beta n}$
   At the beginning we put: $\check{N}_1 = \acute{R}$
   $\check{R}^{\acute{r}}_1 (\acute{r} = 1,2,\ldots,\tau_1)$: The symbol of string related to $x_{t\alpha\beta n}$

$\check{R}^{\acute{r}}_1:$	$\check{R}^{\acute{r}}_{11}$	$\check{R}^{\acute{r}}_{12}$	$\cdots$	$\check{R}^{\acute{r}}_{1\check{N}_1}$

   Which $\check{R}^{\acute{r}}_{1i}$ corresonds to $x^i_{t\alpha\beta n}$, $\quad i = 1,2,\ldots,\acute{R}$
   τ_1 : The number of $\check{R}^{\acute{r}}_1$
   We consider the set of all strings $\check{R}^{\acute{r}}_1$ as follows:

$$\check{U}_1 = \{\check{R}^{\acute{r}}_1 \,|\, \acute{r} = 1,2,\ldots,\tau_1\}$$

   Actually, the members of string $\check{R}^{\acute{r}}_1$ are the numerical values of decision variable $x_{t\alpha\beta n}$, which are randomly selected at the beginning of the work. That is, at the beginning of the work, we have τ_1 strings with $\check{N}_1 = \acute{R}$ members whose members are randomly selected.

2. For y_γ
   $\dot{R} =$ The number y_γ.
   $\check{N} =$ The number of arrays of string related to y_γ

$y_\gamma:$	y^1_γ	y^2_γ	$\cdots$	$y^{\dot{R}}_\gamma$

   At the beginning we put: $\check{N}_2 = \dot{R}$
   $\check{R}^r_2 (r = 1,2,\ldots,\tau_2)$: The symbol of string related to y_γ

$\check{R}^r_2:$	$\check{R}^r_{21}$	$\check{R}^r_{22}$	$\cdots$	$R\,\check{N}^r_2$

   Which $\check{R}^r_{2i}$ corresonds to y^i_γ, $\quad i = 1,2,\ldots,\dot{R}$
   τ_2 : The number of $\check{R}^r_2$
   We consider the set of all strings $\check{R}^r_2$ as follows:

$$\check{U}_2 = \{\check{R}^r_1 \,|\, r = 1,2,\ldots,\tau_2\}$$

   Actually, the members of string $\check{R}^r_2$ are the numerical values of decision variable y_γ, which are randomly selected at the beginning of the work. That is, at the beginning of the work, we have τ_2 strings with $\check{N}_2 = \dot{R}$ members whose members are randomly selected.

$t_{\iota\alpha\beta n}:$	$t^{1}_{\iota\alpha\beta n}$	$t^{2}_{\iota\alpha\beta n}$	$\cdots$	$t^{\acute{S}}_{\iota\alpha\beta n}$

3. For $t_{\iota\alpha\beta n}$
   $\acute{R}=$ The number of $t_{\iota\alpha\beta n}$

   $\check{N}_3=$ The number of arrays of string related to $t_{\iota\alpha\beta n}$
   At the beginning we put: $\check{N}_3=\acute{S}$
   $\check{R}^{\acute{r}}_3(\acute{r}=1,2,\ldots,\tau_3)$: The symbol of string related to $t_{\iota\alpha\beta n}$

$\check{R}^{\acute{r}}_3:$	$\check{R}^{\acute{r}}_{31}$	$\check{R}^{\acute{r}}_{32}$	$\cdots$	$\check{R}^{\acute{r}}_{3\check{N}_3}$

   Which $\check{R}^{\acute{r}}_{3i}$ corresonds to $t^{i}_{\iota\alpha\beta n}$, $\quad i=1,2,\ldots,\acute{S}$
   τ_3 : The number of $\check{R}^{\acute{r}}_3$
   We consider the set of all strings $\check{R}^{\acute{r}}_3$ as follows:

$$\check{U}_3=\{\check{R}^{\acute{r}}_3\,|\,\acute{r}=1,2,\ldots,\tau_3\}$$

   Actually, the members of string $\check{R}^{\acute{r}}_3$ are the numerical values of decision variable $t_{\iota\alpha\beta n}$, which are randomly selected at the beginning of the work. That is, at the beginning of the work, we have τ_3 strings with $\check{N}_3=\acute{S}$ members whose members are randomly selected.

4. For $\pi_{\iota\alpha\beta n}$
   $\hat{S}=$ The number of $\pi_{\iota\alpha\beta n}$

$\pi_{\iota\alpha\beta n}:$	$\pi^{1}_{\iota\alpha\beta n}$	$\pi^{2}_{\iota\alpha\beta n}$	$\cdots$	$\pi^{\hat{S}}_{\iota\alpha\beta n}$

   $\check{N}_4=$ The number of arrays of string related to $\pi_{\iota\alpha\beta n}$
   At the beginning we put: $\check{N}_4=\hat{S}$
   $\check{R}^{\acute{r}}_4(\acute{r}=1,2,\ldots,\tau_4)$: The symbol of string related to $\pi_{\iota\alpha\beta n}$

$\check{R}^{\acute{r}}_4:$	$\check{R}^{\acute{r}}_{41}$	$\check{R}^{\acute{r}}_{42}$	$\cdots$	$\check{R}^{\acute{r}}_{4\check{N}_4}$

   Which $\check{R}^{\acute{r}}_{4i}$ corresonds to $\pi^{i}_{\iota\alpha\beta n}$, $\quad i=1,2,\ldots,\hat{S}$
   τ_4 : The number of $\check{R}^{\acute{r}}_4$
   We consider the set of all strings $\check{R}^{\acute{r}}_4$ as follows:

$$\check{U}_4=\{\check{R}^{\acute{r}}_4\,|\,\acute{r}=1,2,\ldots,\tau_4\}$$

   Actually, the members of string $\check{R}^{\acute{r}}_4$ are the numerical values of decision variable $\pi_{\iota\alpha\beta n}$, which are randomly selected at the beginning of the work. That is, at the beginning of the work, we have τ_4 strings with $\check{N}_4=\acute{S}$ members whose members are randomly selected.

5. For t_γ
   $\check{S}=$ The number of t_γ

$t_\gamma:$	t^{1}_{γ}	t^{2}_{γ}	$\cdots$	$t^{\check{S}}_{\gamma}$

   At the beginning we put: $\check{N}_5=\check{S}$ $\check{N}_5=$ The number of arrays of string related to t_γ
   $\check{R}^{\acute{r}}_5(\acute{r}=1,2,\ldots,\tau_5)$: The symbol of string related to t_γ

$\check{R}^{\acute{r}}_5:$	$\check{R}^{\acute{r}}_{51}$	$\check{R}^{\acute{r}}_{52}$	$\cdots$	$\check{R}^{\acute{r}}_{5\check{N}_5}$

Which $\check{R}^{\acute{r}}_{5i}$ corresonds to t^{i}_{γ}, $\qquad i = 1,2,\ldots,\check{S}$
τ_5 : The number of $\check{R}^{\acute{r}}_{5}$
We consider the set of all strings $\check{R}^{\acute{r}}_{5}$ as follows:

$$\check{U}_5 = \{\check{R}^{\acute{r}}_{5} \,|\, \acute{r} = 1,2,\ldots,\tau_5\}$$

Actually, the members of string $\check{R}^{\acute{r}}_{5}$ are the numerical values of decision variable t_{γ}, which are randomly selected at the beginning of the work. That is, at the beginning of the work, we have τ_5 strings with $\check{N}_5 = \check{S}$ members whose members are randomly selected.

6. For $t^{'}_{t\alpha\beta n}$
$\d{S}$ = The number of $t^{'}_{t\alpha\beta n}$

$t^{'}_{t\alpha\beta n}$:	$t^{'1}_{t\alpha\beta n}$	$t^{'2}_{t\alpha\beta n}$	$\cdots$	$t^{'\d{S}}_{t\alpha\beta n}$

At the beginning we put: $\check{N}_6 = \d{S}$
$\check{N}_6$ = The number of arrays of string related to $t^{'}_{t\alpha\beta n}$
$\check{R}^{\acute{r}}_{6}(\acute{r} = 1,2,\ldots,\tau_6)$: The symbol of string related to $t^{'}_{t\alpha\beta n}$

$\check{R}^{\acute{r}}_{6}$:	$\check{R}^{\acute{r}}_{61}$	$\check{R}^{\acute{r}}_{62}$	$\cdots$	$\check{R}^{\acute{r}}_{6\check{N}_6}$

Which $\check{R}^{\acute{r}}_{6i}$ corresonds to $t^{'i}_{t\alpha\beta n}$, $\qquad i = 1,2,\ldots,\d{S}$
τ_6 : The number of $\check{R}^{\acute{r}}_{6}$
We consider the set of all strings $\check{R}^{\acute{r}}_{6}$ as follows:

$$\check{U}_6 = \{\check{R}^{\acute{r}}_{6} \,|\, \acute{r} = 1,2,\ldots,\tau_6\}$$

Actually, the members of string $\check{R}^{\acute{r}}_{6}$ are the numerical values of decision variable $t^{'}_{t\alpha\beta n}$, which are randomly selected at the beginning of the work. That is, at the beginning of the work, we have τ_6 strings with $\check{N}_6 = \d{S}$ members whose members are randomly selected.

The main string of the problem, which corresponds to all the decision variables, is the combination of these six strings.

$\check{U}$:Main string	$\check{U}_1$	$\check{U}_2$	$\check{U}_3$	$\check{U}_4$	$\check{U}_5$	$\check{U}_6$

It is clear that the length of the main string is equal to $\check{N} = \sum_{i=1}^{6} \tau_i$.
Now we construct the main and complete string $\check{U}$ which corresponds to the decision variables of our designed problem, as follows.

$$\check{U} = \bigcup_{i=1}^{6} \check{U}_i == \{U_n \,|\, n = 1,2,\ldots,\check{N}\}$$

Now we arrange the members of $\check{U}$ and display it as a vector.

$$W = (W_1, W_2, \ldots, W_{\check{N}})$$

By designing a random number generator, we generate I number of samples of the vector W. Suppose the generated samples are $V_1, V_2, \ldots, V_I$. Now we put the generated samples in the vector V.

$$V = \{V_i \,|\, i = 1,2,\ldots,I\}$$

Since our problem is restricted, then for any given vector, the algorithm must first check the feasibility and then fit from the optimal point of view. Therefore, after obtaining the feasible vectors, the process of generation and mutation is performed.

Now we define the formula of generation of new chromosome. Suppose x and y are two chromosomes that randomly selected, the new chromosome is generated by the following formulas:

$$z_1 = \frac{x+y}{2}, \qquad z_2 = \sqrt{xy}$$

This new chromosome is added to the string of previous chromosomes. That is, in fact, the algorithm always increases the length of the chromosomal string until the length of the string reaches the predetermined limit. When the length of the chromosomal strand reaches a predetermined limit, the mutation process takes place. The mutation process is done using the evaluator function. The mutation function is defined according to the objective function. The evaluator function assigns a number to each chromosome. The algorithm sorts the assigned numbers and selects the best chromosomes from among them equal to the initial length of the string and forms a new string. It is quite obvious that the new string is not worse than the previous string. Therefore, the algorithm moves towards better solution. To terminate the iterations of the algorithm, either we define a stop condition or determine the number of iterations at the beginning. Now we define the evaluator function for each string $\breve{U}$ as follows:

$$
\begin{aligned}
&\Psi = \Psi(V) = \Psi(\breve{U}) = \Psi(\breve{U}_1, \breve{U}_2, \breve{U}_3, \breve{U}_4, \breve{U}_5, \breve{U}_6) \\
&= \Psi(\breve{R}_1^{\acute{r}}, \breve{R}_2^{\acute{r}}, \breve{R}_3^{\acute{r}}, \breve{R}_4^{\acute{r}}, \breve{R}_5^{\acute{r}}, \breve{R}_6^{\acute{r}}) = \Psi(x_{t\alpha\beta n}, y_\gamma, t_{t\alpha\beta n}, \pi_{t\alpha\beta n}, t_\gamma, t'_{t\alpha\beta n}) \\
&\sum_{i=1}^{6} C_i = \sum_{i=1}^{6} F_i + \sum_{\sigma=1}^{6} G_\sigma + \sum_{\theta=1}^{11} W_\theta + \sum_{\beta=1}^{10} T_\beta + \sum_{\gamma\in\Omega} t_\gamma + \sum_{\delta=1}^{10} T'_\delta \\
&= \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} c_{tkjn}x_{tkjn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} c_{tjqn}x_{tjqn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} c_{tjen}x_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} c_{tjbn}x_{tjbn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} c_{tjrn}x_{tjrn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} c_{tqbn}x_{tqbn} + \\
&\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} c_{tqen}x_{tqen} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} c_{tqrn}x_{tqrn} + \\
&\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} c_{tbrn}x_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} c_{tben}x_{tben} + \\
&\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} w_{tmkn}x_{tmkn} + \sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} w_{tkjn}x_{tkjn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} w_{tjqn}x_{tjqn} + \sum_{t=1}^{N}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} w_{tjen}x_{tjen} + \\
&\sum_{t=1}^{N}\sum_{j=1}^{N}\sum_{b=1}^{B}\sum_{n=1}^{N} w_{tjbn}x_{tjbn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} w_{tjrn}x_{tjrn} + \\
&\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} w_{tqen}x_{tqen} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} w_{tqbn}x_{tqbn} + \\
&\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} w_{tqrn}x_{tqrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} w_{tbrn}x_{tbrn} + \\
&\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} w_{tben}x_{tben} + \sum_{i=1}^{K} c_i^k y_i^k + \sum_{j=1}^{J} c_i^j y_i^j + \\
&\sum_{i=1}^{E} c_i^e y_i^e + \sum_{i=1}^{Q} c_i^q y_i^q + \sum_{i=1}^{B} x_i^b y_i^b + \sum_{i=1}^{R} c_i^r y_i^r + \sum_{\gamma\in\Omega} t_\gamma \\
&\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} t_{tkjn}\pi_{tkjn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} t_{tjqn}\pi_{tjqn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tjen}\pi_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tjbn}\pi_{tjbn} + \\
&\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tjrn}\pi_{tjrn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tqbn}\pi_{tqbn} +
\end{aligned}
$$

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tqen}\pi_{tqen} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tqrn}\pi_{tqrn} +$$
$$\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tbrn}\pi_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tben}\pi_{tben} +$$
$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tjen}\pi_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tjbn}\pi_{tjbn} +$$
$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tjrn}\pi_{tjrn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} t_{tqbn}\pi_{tqbn} +$$

$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tqen}\pi_{tqen} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tqrn}\pi_{tqrn} +$$
$$\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} t_{tbrn}\pi_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} t_{tben}\pi_{tben} +$$
$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{n=1}^{N} t'_{tkjn}x_{tkjn} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{q=1}^{Q}\sum_{n=1}^{N} t'_{tjqn}x_{tjqn} +$$
$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tjen}x_{tjen} + \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{b=1}^{B}\sum_{n=1}^{N} t'_{tjbn}x_{tjbn} +$$
$$\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tjrn}x_{tjrn} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{b=1}^{B}\sum_{n=1}^{N} t'_{tqbn}x_{tqbn} +$$
$$\sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tqen}x_{tqen} + \sum_{t=1}^{T}\sum_{q=1}^{Q}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tqrn}x_{tqrn} +$$
$$\sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{r=1}^{R}\sum_{n=1}^{N} t'_{tbrn}x_{tbrn} + \sum_{t=1}^{T}\sum_{b=1}^{B}\sum_{e=1}^{E}\sum_{n=1}^{N} t'_{tben}x_{tben} +$$

Now we have to do two things. First, generate a number of feasible solutions and then use them to generate new solutions to reach a suitable approximate solution.

According to the above, we can now describe the steps of the algorithm.

Algorithm 1: Genetic Algorithm to find feasible solutions

Step 1. Begin

Step 2. Read $\check{N}$

Step 3. Read the initial value of V

Step 4. Put $\Delta = 0$

Step 5. Put $F = \nabla\phi$ Step 6. Choose two random numbers in range $[1, \check{N}]$. Suppose the two numbers are α and β. In this case, generate the two new chromosomes as follows:

$$z_1^{\alpha,\beta} = \frac{v_\alpha + v_\beta}{2}, \qquad z_2^{\alpha,\beta} = \sqrt{v_\alpha v_\beta}$$

Step 7. If $z_1^{\alpha,\beta}$ is feasible, then go to step 9, Otherwise, go to step 8.

Step 8. If $z_2^{\alpha,\beta}$ is feasible, then go to step 10, Otherwise, go to step 6.

Step 9. Put $F = F \cup \{z_1^{\alpha,\beta}\}, \Delta = \Delta + 1$ and go to step 11.

Step 10. Put $F = F \cup \{z_2^{\alpha,\beta}\}, \Delta = \Delta + 1$ and go to step 11.

Step 11. If $\Delta = \check{N}$ then go to step 12, Otherwise, go to step 6.

Step 12. End.

Algorithm 2: Genetic algorithm to find optimal feasible solutions

Step 1. Begin

Step 2. Put the initial value of $F, \nabla, \check{N}$
Step 3. $H = \emptyset$
Step 4. $\bar{E} = F\check{N}$
Step 5. $\Delta = 0$
Step 6. Read the initial value of V
Step 7. $k = 1$
Step 8. Choose two random numbers in range $[1, \check{N}]$. Suppose the two numbers are α and β. In this case, generate the two new chromosomes as follows:

$$z_1^{\alpha,\beta} = \frac{v_\alpha + v_\beta}{2}, \qquad z_2^{\alpha,\beta} = \sqrt{v_\alpha v_\beta}$$

Step 9. If $z_1^{\alpha,\beta}$ is feasible, then go to step 11, Otherwise, go to step 10.
Step 10. If $z_2^{\alpha,\beta}$ is feasible, then go to step 12, Otherwise, go to step 8.
Step 11. Put $F = F \cup \{z_1^{\alpha,\beta}\}, \Delta = \Delta + 1$ and go to step 13.
Step 12. Put $F = F \cup \{z_2^{\alpha,\beta}\}, \Delta = \Delta + 1$ and go to step 13.
Step 13. If $\Delta > \bar{E}$ then go to step 14, Otherwise, go to step 8.
Step 14. $J = 1$
Step 15. $G = F$
Step 16. $I = 1$
Step 17. Select a member of set G (for example $G^{\approx}$) and put $\Psi_{\min} = \Psi(G^{\approx})$ and go to the next step.
Step 18. Put $G = G - \{G^{\approx}\}$ and go to next step.
Step 19. Select a member of set G (for example $G^{\approx}$) and go to the next step.
Step 20. If $\Psi(G^{\approx}) < \Psi_{\min}$ then $\Psi_{\min} = \Psi(G^{\approx})$ and go to step 21, otherwise go to step 21.
Step 21. $I = I + 1$
Step 22. If $I \leq \Delta$ go to step 17, otherwise put $\Psi(G^*) = \Psi_{\min}$ and go to next step.
Step 23. Put $H = H \cup \{G^*\}$
Step 24. $J = J + 1$
Step 25. If $J > \check{N}$ go to next step, otherwise go to step 15.
Step 26. Place the members of H in vector of V and go to next step.
Step 27. $k = k + 1$
Step 28. If $k > \nabla$ go to next step, otherwise go to step 8.
Step 29. End.

We will implement this algorithm in the next chapter and solve an example of the designed problem and extract numerical results.

6 Implementing Algorithms and Solving Numerical Examples

In this section, using MATLAB software, we implement the algorithms of the previous chapter and solve the numerical example.

Example 1. *Consider an inverse recycling network with the following data.*

$$M = 20,\ K = 6,\ J = 5,\ Q = 3,\ E = 5,\ B = 3,\ R = 2,\ t = 4,\ n = 4.$$
$$\nu = M + K + J + Q + E + B + R = 100 + 10 + 5 + 3 + 5 + 3 + 2 = 41$$
$$\varepsilon = MK + KJ + JQ + JE + JB + JR + QR + +QE + QB + BE + BR = 200$$

With the above data, the network has 41 *nodes and* 200 *edges. So, this relatively small example shows that the complexity of the designed problem is not good.*

Now we get the rest of the data.

$t_{\max}$: The maximum amount of time that can be allocated to complete all network activities $= 300000$

$t_\gamma^{\max}$: The maximum time required to build the center $\gamma = 5500$ d_1: The percentage of commodities transferred from collection centers

Table 1. S_t : The amount of commodity of t used by consumers

t	1	2	3	4
S_t	150	234	436	135

Table 2. c_k : The cost of building a local collection center of k

k	1	2	3	4	5	6
c_k	190	285	200	100	155	169

Table 3. c_j : The cost of building a collection center of j

j	1	2	3	4	5
c_j	450	345	226	365	300

Table 4. c_q : The cost of building a repair center of q

q	1	2	3
c_q	655	568	476

Table 5. c_e : The cost of building a charity collection center of e

e	1	2	3	4	5
c_e	266	324	150	432	221

Table 6. c_b : The cost of building a recycling center of b

b	1	2	3
c_b	432	112	236

Table 7. c_r : The cost of building a disposal center of r

r	1	2
c_r	325	451

Table 8. z_{tk} : The capacity of local collection center of k for commodity of t

t,k	1	2	3	4	5	6
1	2500	3100	2034	2200	4050	3900
2	4320	3478	5468	45872	3689	3458
3	3459	6578	65890	3468	3478	2500
4	2367	1458	45678	5780	6543	5643

Table 9. z_{tj} : The capacity of collection center of j for commodity of t

t,j	1	2	3	4	5
1	1324	6733	6512	34567	3487
2	3465	4576	3467	765	347
3	2567	9822	345	4589	870
4	378	5467	67891	651	2854

to repair centers

d_2: The percentage of commodities transferred from collection centers to charity centers

Table 10. z_{tq} : The capacity of charity center of q for commodity of t

t,q	1	2	3
1	9073	24	6347
2	456	5683	5981
3	56976	347	35278
4	453	597	376

Table 11. z_{te} : The capacity of charity center of e for commodity of t

t,e	1	2	3	4	5
1	3651	9833	487	57623	4652
2	209	7862	762	342	3875
3	45097	4683	458776	109	9864
4	5683	3476	6932	6783	56733

Table 12. z_{tb} : The capacity of recycling center of b for commodity of t

t,b	1	2	3
1	4587	345	4598
2	6788	45622	7864
3	8887	465	3568
4	653	1002	98234

Table 13. z_{tr} : The capacity of disposal center of r for commodity of t

t,r	1	2
1	3687567	547
2	45698	5698
3	246	7643
4	6790	2387

Table 14. c_{tk} : The cost of collecting of a unit commodity of t at the local collection center of k

t,k	1	2	3	4	5	6
1	2654	6578	1239	1763	5684	8654
2	4598	609	543	3456	24644	4590
3	56799	346	8744	576	247	6512
4	652	4577	349	9435	2870	1116

Table 15. c_{tj} : The cost of collecting of a unit commodity of t at the collection center of j

t,j	1	2	3	4	5
1	2567	457	567	23098	9843
2	6789	5432	6521	8734	6523
3	7654	4587	9823	690	5687
4	6543	6587	332	2133	679

d_3: The percentage of commodities transferred from collection centers to recycling centers

d_4: The percentage of commodities transferred from collection centers to disposal centers

Table 16. z_{tq} : The cost of repair a unit of commodity of t in the repair center q

t,q	1	2	3
1	8734	126	9076
2	4576	156	6587
3	677	6543	8734
4	37644	446	7654

Table 17. c_{tr} : The cost of disposal a unit of commodity of t in the disposal center r

t,r	1	2
1	5687	5687
2	7654	76656
3	1111	9967
4	3245	7689

Table 18. c_{tb} : The cost of recycling a unit of commodity of t in the recycling center b

t,b	1	2	3
1	254	56788	398
2	34	687	6523
3	3467	9043	1432
4	764	21	3984

Table 19. $e_\tau(\tau = m,k,j,q,e,b,r)$: The total of commodities entering node τ minus the total of commodities leaving node τ

τ	m	k	j	q	e	b	r
e_τ	27644	4875	34908	456	3965	60912	4598

Table 20. c_{tmjn} : The cost of transporting a unit commodity t from the consumer m to the collection center j by vehicle n. $t = 1$ and $n = 1$

m,j	1	2	3	4	5
1	2376	3654	9811	345	1349
2	4876	4355	3876	4576	6532
3	2765	9843	8966	5432	7832
4	365	45687	5678	234	4578
5	6798	5432	7622	224	873
6	5687	1654	4326	3467	3476
7	9032	1324	5673	3456	7834
8	344	4356	5432	34988	7654
9	3456	6578	1655	543	567

6.1 Implementing Algorithms and Solving Numerical Examples

With the above data, we have implemented the algorithms using MATLAB software. The outputs (Results related to implementation of MATLAB software), which are actually the optimal (almost optimal) values of the decision variables, are reported in the following tables.

Table 21. w_{tmkn} : The cost of collection (in the local collection center of k) of a unit of commodity of t sent from the center of m to the center of k by vehicle of n. $t = 1$ and $n = 1$

m,k	1	2	3	4	5	6
1	3657	3683	2769	37688	6573	23567
2	5632	2365	6789	5673	3546	7893
3	987	458	102	3879	3879	2456
4	2347	44569	467	479	5478	5678
5	891	35468	5678	792	5782	3657
6	2347	45632	57903	2568	2768	3276
7	78902	4889	5899	4698	5587	57611
8	54678	3765	6879	4766	35447	12009
9	4798	3567	6754	5679	5476	6578

Table 22. Values of $d_i (i = 1,2,3,4)$

d_i	d_1	d_2	d_3	d_4
Percentage	0.37	0.46	0.14	0.3

Table 23. z_n: Vehicle capacity n

z_n	z_1	z_2	z_3	z_4
Vehicle capacity n	20	10	2	1

Table 24. v_t: Commodity volume of t

v_t	v_1	v_2	v_3	v_4
Commodity volume of t	120	145	250	300

Table 25. x_{tkjn}: Amount of transportation of commodity of t from local collection center of k to collection center of j by vehicle of n. $t = 1$ and $n = 1$

k, j	1	2	3	4	5
1	34.21	54.78	65.90	76.54	45.78
2	54.32	34.67	65.98	76.32	78.90
3	32.00	43.90	54.90	32.11	76.80
4	65.90	54.33	53.20	21.09	65.32
5	54.89	48.79	54.78	20.06	35.46
6	65.33	55.48	32.90	65.90	47.78

7 Conclusions and Future Work

In this research, we have examined the issue of the recycling process of used commodities. We have divided the used commodities into four categories. Usable commodities, commodities that must be repaired and used, commodities that can be used after recycling, and commodities that cannot be used at all and must be disposed of. First category commodities should be sent directly to consumption centers (charity centers). Second category commodities, first are sent to repair centers and after repair they are sent to charity centers. Some of them that cannot be repaired are sent to recycling centers or disposal centers. Third category commodities are sent directly to recycling centers and those that can be recycled are sent to charity centers after recycling and the rest are sent to disposal centers. The fourth category commodities are sent directly to disposal centers. The objectives of this research are to minimize the cost of transporting commodities, minimize the cost of building centers, minimize the costs of operations (collection, repair, recycling and disposal) in centers, minimize the

Table 26. y_i^{μ} : The defining variable of the construction i^{th} of the center of μ. $y_i^{\mu} = \begin{cases} 1, & If i^{th} of the center of \mu is built \\ 0, & Otherwise \end{cases}$

μ, i	1	2	3	4	5	6	7	8	9
m	1	1	0	1	1	1	1	1	1
k	1	1	1	1	0	1	−−−	−−−	−−−
i	0	1	1	1	1	−−−	−−−	−−−	−−−
q	1	1	1	−−−	−−−	−−−	−−−	−−−	−−−
e	1	1	1	1	1	−−−	−−−	−−−	−−−
b	1	1	1	−−−	−−−	−−−	−−−	−−−	−−−
r	1	1	−−−	−−−	−−−	−−−	−−−	−−−	−−−

Table 27. t_i^{γ} : Time needed to built the i^{th} center $\gamma, \gamma \in \Omega$

γ, i	1	2	3	4	5	6	7	8	9
m	1578	135487	9870	1469	1873	11156	91022	2169	3381
k	712091	54710	1274	56901	55056	14590	−−−	−−−	−−−
i	33024	38136	1653	1259	4109	−−−	−−−	−−−	−−−
q	55891	19369	17338	−−−	−−−	−−−	−−−	−−−	−−−
e	12390	13458	44109	22981	52190	−−−	−−−	−−−	−−−
b	1345	1986	1	−−−	−−−	−−−	−−−	−−−	−−−
r	4571	7188	−−−	−−−	−−−	−−−	−−−	−−−	−−−

Table 28. t_{tjen}: The time required to transfer (one transfer) of commodity of t from the collection center of j to the charity center of e by vehicle of n. $t = 1, n = 1$

j, e	1	2	3	4	5
1	65.47	58.79	55.98	76.98	98.32
2	47.68	4.65	43.78	65.98	56.57
3	36.58	0.55	87.98	67.82	65.78
4	43.76	5.46	66.98	17.68	65.78
5	0.87	0.66	45.67	5.47	8.76

Table 29. π_{tjen}: The number of transfers that vehicle of n needs to carry commodity of t from center of j to center of e. $t = 1, n = 1$

j, e	1	2	3	4	5
1	4.32	4.73	7.35	3.73	5.56
2	2.18	7.70	8.71	8.95	5.88
3	1.15	8.54	8.32	9.55	5.84
4	2.47	5.30	8.12	3.65	6.61
5	3.44	2.64	2.43	9.97	8.96

time of moving commodities, minimize the time of building centers and minimize the time of the activities in the centers. In this thesis, first, by understanding the conditions of the problem, we have modeled it. The problem model became a nonlinear multi-objective model. Due to the complexity of the model, we solved it by genetic algorithm method. The implementation of the method led us to the conclusion that the problem is very hard in general and solving it in large sizes requires a lot of cost and time. Also, in the analysis of the model and method, we found the following suggestions that can be the subject of new research in the future.

- Using other methods (other methods for solving multi-objective problems) to solve the multi-objective model of this research.

- Using other meta-heuristic methods.
- Using artificial intelligence and neural networks.
- Using fuzzy models.

Authors' Contributions

All authors have the same contribution.

Data Availability

The manuscript has no associated data or the data will not be deposited.

Conflicts of Interest

The authors declare that there is no conflict of interest.

Ethical Considerations

The authors have diligently addressed ethical concerns, such as informed consent, plagiarism, data fabrication, misconduct, falsification, double publication, redundancy, submission, and other related matters.

Funding

This research did not receive any grant from funding agencies in the public, commercial, or nonprofit sectors.

Acknowledgments

The authors thank the editor and reviewers for their helpful comments.

References

[1] A. Aliyari Boroujeni, M.R. Ghaemi, R. Pourgholi, Solving the Transportation Problem using Meta-Heuristic Algorithms, Analytical and Numerical Solutions for Nonlinear Equations, 9(1), 12–19, (2024).

[2] F. Altiparmak, M. Gen, L. Lin, T. Paksoy, A genetic algorithm approach for multi-objective optimization of supply chain networks, Computers and Industrial Engineering, 51, 197–216, (2006).

[3] A. Amiri, Designing network in a supply chain system: Formulation and efficient solution procedure, European Journal of Operational Research, 171, 567–576, (2006).

[4] A. Anjomshoae, A. Hassan, K.Y. Wong, An integrated AHP-based scheme for performance in humanitarian supply chains, International Journal of Productivity and Performance Management, 68(5), 938–957, (2019).

[5] G.H. Askari Robati, A. Hashemi Borzabadi, A. Heydari, Pareto-optimal Solutions for multiobjective optimal control problems using hybrid IWO/PSO algorithm, Analytical and Numerical Solutions for Nonlinear Equations (Global Analysis and Discrete Mathematics), 4(1), 41–60, (2019).

[6] A.A. Bani, M. Darbani, A Method for Solving Optimal Control Problems Using Genetic Programming, Analytical and Numerical Solutions for Nonlinear Equations (Global Analysis and Discrete Mathematics), 2(2), 103–114, (2018).

[7] G. Barbarosoglu, O. Linet, A. Cevik, An interactive approach for hierarchical analysis of helicopter logistics in disaster relief operations, European Journal of Operational Research, 140, 118–133, (2002).

[8] W. Chen, Y. Liu, M. Han, Designing a sustainable reverse logistics network for used cell phones based on offline and online trading systems, J. Environ. Manage., 354:120417, (2024).

[9] T.H. Cormen, C.E. Leiserson, R.L. Rivest, C. Stein, Introduction to Algorithms (Third Edition), The MIT Press, ISBN:978-0-262-03384-8, (2009).

[10] H. Dana Mazraeh, P. Motaharinezhad, N. Daneshian, K. Parand, Solving the Fokker-Planck Equation with Neural Networks: A Performance Improvement Approach, Analytical and Numerical Solutions for Nonlinear Equations, 9(1), 1–11, (2024).

[11] J. Dethloff, Vehicle routing and reverse logistic: the vehicle routing problem with simultaneous delivery and pick-up, OR Spektrum, 23(1), 79–96, (2001).

[12] R.G. Dondo, C.A. Mendez, Operational planning of forward and reverse logistic activities on multi-echelon supply-chain networks, Computers and Chemical Engineering, 88, 170–194, (2016).

[13] F. Du, G.W. Evans, A bi-objective reverse logistics network analysis for post-sale service, Computers and Operational Research, 35, 2617–2534, (2008).

[14] M.A. Ertem, N. Buyurgan, M.D. Rossetti, Multiple buyer procurement auctions framework for humanitarian supply chain management, International Journal of Physical Distribution and Logistics Management, 40(3), 202–227, (2010).

[15] G. Fuyang, Design of Reverse Logistics Network for Power Battery Considering Transportation Risk, International Journal of Computer Science and Information Technology, 3(3), (2024).

[16] V. Jayaraman, H. Pirkul, Planning and coordination of production and distribution facilities for multiple commodities, European Journal of Operational Research, 133, 394–408, (2001).

[17] H. Kim, J. Yang, K.D. Lee, Vehicle routing in reverse logistics for recycling end-of-life consumer electronic goods in South Korea, Transportation Research Part D: Transport and Environment, 14(5), 291–299, (2009).

[18] H.S. Kilic, U. Cebeci, M.B. Ayhan, Reverse logistics system design for the waste of electrical and electronic equipment (WEEE) in Turkey, Resources, Conservation and Recycling, 95, 120–132, (2015).

[19] H.J. Ko, G.W. Evans, A genetic-based heuristic for the dynamic integrated forward/reverse logistics network for PLS, Computers and Operations Research, 34, 346–366, (2007).

[20] M. Kucuk, Y.S. Topaloglu, Constraint programming-based solution approaches for three-dimensional loading capacitated vehicle routing problems, Computers and Industrial Engineering, 171, 108505, (2022).

[21] Z. Lan, D. Tao, L. Zhuofan, Reverse Logistics Network Design for Medical Waste Disposal under the Scenario of Uncertain Proposal Demand, Sustainability, MDPI, 16(7), 1–14, (2024).

[22] D. Lee, M. Dong, A heuristic approach to logistics network design for end-of-lease computer products recovery, Transportation Research, Part E, 44, 455–474, (2007).

[23] J. Lee, E.M. Gen, K.G. Rhee, Network model and optimization of reverse logistics by hybrid genetic algorithm, Computers and Industrial Engineering, 56(3), 951–964, (2009).

[24] G. Li, J. Liu, A. Giordano, Robust optimization of construction waste disposal facility location considering uncertain factors, Journal of Cleaner Production, 353, 131455, (2022).

[25] H. Li, S.A. Alumur, Multi-period reverse logistics network design for water resource management in hydraulic fracturing, Applied Mathematical Modelling, 129, 612–632, (2024).

[26] E. Melachrinoudis, A. Messac, H. Min, Consolidating a warehouse network: a physical programming approach, International Journal of Production Economics, 97, 1–7, (2005).

[27] L. Messmann, S. Boldoczki, A. Thorenz, A. Tuma, Potentials of preparation for reuse: A case study at collection points in the German state of Bavaria, Journal of Cleaner Production, 211, 1534–1546, (2019).

[28] H. Min, H.J. Ko, C.S. Ko, A genetic algorithm approach to developing the multi-echelon reverse logistics network for product returns, Omega, 34(1), 56–69, (2006).

[29] J. Shi, G. Zhang, J. Sha, Optimal production planning for a multi-product closed loop system with uncertain demand and return, Computers and Operational Research, 38(3), 641–650, (2011).

[30] G.H. Shirdel, M. Abdolhosseinzadeh, Markov Chain Anticipation for the Online Traveling Salesman Problem by Simulated Annealing Algorithm, Analytical and Numerical Solutions for Nonlinear Equations (Global Analysis and Discrete Mathematics), 2(1), 33–38, (2017).

[31] S. Xu, Y. Hao, A two-level decision-support framework for reverse logistics network design considering technology transformation in Industry 4.0: a case study in Norway, The International Journal of Advanced Manufacturing Technology, 134, 1–2, (2024).