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Forecasting Palladium Price Using GM(1,1)

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Abstract Palladium is an element of PGM group that has significant physical properties. This leads to more attention to this metal. Due to vast applications of palladium in industry and its usage in jewelry, its price plays an important role in economic. Therefore, forecasting its price is crucial subject in economic and engineering design. This paper proposes the model GM(1,1) to predict the Palladium price. The method is applied in experimental data for two recent years. The results show that the method is robust and accurate.

Keywords Grey system · Platinium groups · Time series

Mathematics Subject Classification (2010) 6207 · 62M20 · 65L09 · 62M10

1 Introduction

Palladium is the element with atomic number 46 and has the chemical symbol Pd. It is called after the asteroid Pallas. Palladium, platinum, ruthenium, iridium, osmium and rhodium are part of the group of elements referred to as the platinum group metals (PGMs). The elements in these group have similar properties like high catalytic and resistant qualities. The special characteristics

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of the PGMs have caused their widespread usage in industry. Recently, PGMs are used in more that 25% of industrial products.

Physically, Palladium can be described as a soft silver-white metal that resembles platinum. It is the least dense and has the lowest melting point in PGM group. It is soft and ductile when annealed and is greatly increased in strength and hardness when cold-worked. Palladium does not react with oxygen at standard temperatures (and thus does not tarnish in air) [1,3].

Palladium is being used in a number of different applications, below is a short list of them.

1.1 Catalysis

When palladium is finely divided on carbon, the results would be a versatile catalyst. A palladium catalyst can speed up (de)hydrogenation and petroleum cracking. Dispersion of palladium on a conductive material would work as a very good electro catalyst for oxidizing primary alcohols [8]. As a homogeneous catalyst, palladium can be used for highly sensitive and selective chemical transformation [5,9].

1.2 Jewelry

Palladium has been used in jewelry since the 1930s as an alternative to platinum for making white gold, because of its natural metallic white color. Palladium is much lighter than platinum and just like gold, it can be made into a 100nm thin form [3].

White gold is made as an alloy of gold which is usually consisted of three metals: nickel, silver and palladium. The palladium - gold alloy is more expensive than nickel - gold, but it may cause an allergic reaction in some people [5].

Because of the relative rise in price of gold and platinum in recent years, palladium has been used as the principal metal in jewelry [18]. China for example, began fabricating significant amount of palladium jewelry which even the relative decline of the price of platinum has not stopped it and its total production was 17.4 tons as of 2009 [15,14].

1.3 Electronics

One of the biggest applications of palladium is in electronics industry, especially in the production of capacitors [5,17]. Palladium is used as an electrode in this section. Connector plating in consumer electronics also require amounts of palladium, which sometimes is used as palladium nickel alloy [16,2]. The largest area of palladium use in the electronics sector is in multi-layer ceramic (chip) capacitors (MLCC) [4,20]. Smaller amounts of palladium are used in conductive tracks in hybrid integrated circuits (HIC) and for plating connectors and lead frames.

1.4 Trading

Platinum and palladium have a very recent history, unlike gold and silver, which have been known since the earliest civilizations. Platinum was categorized as a precious metal in 1751 and palladium was isolated as a separate metal less than 200 years ago. In this not so long period and despite only limited availability of the mentioned metals, they have made major contributions to modern scientific progress.

Palladium trading takes place on Tokyo Commodity Exchange (TOCOM), London Metal Exchange (LME) and New York Mercantile Exchange (NYMEX).

Price of palladium is set by the London Platinum and Palladium Market (LPPM), which bases its price on the direct trading activities on their platform.

Each day, four LPPM members (large international banks, producers, refiners, fabricators, manufacturers and distributors) fix the bid prices on palladium twice. The bid price is one at which members of LPPM guarantee that they will buy good delivery metal [19].

1.5 Price Factors

Like most of the industries and goods, the price of palladium is discovered through the supply and demand chain. Although palladium is considered a precious metal, its price is influenced mainly by its demand in the automobile industry. So studying the movements in this industry can prove a significant indicator for future palladium prices. The supply also plays a major role in the price of palladium discovery as a large quantity of the metal is produced in South-Africa, where it is produced cheaper in contrast to the dollar in which it is eventually priced [21].

The price of several metals has traditionally been connected with each other. For example, gold and silver prices are looked at in isolation and relative to each other, mostly because both metals play a major role in jewelry trade. Like gold and silver, less prominent platinum and palladium can be both mined and used in very similar applications. The Platinum Group Metals, or PGMs, are often magmatic in origin and rare in economic concentrations. As mentioned before, the majority of the worlds platinum and palladium comes from South Africa, Zimbabwe and Russia. But, recently the low-cost surface mines have given way to deep, expensive and complex operations.

For the time being palladium has come out of the shadows and is having its day in the spotlight. Therefore, the prediction of palladium price is crucial subject in economic, manufacturing and jewelry business. In this paper, the palladium price is predicted using grey system theory. The rest of the paper is organized as follows.

In Section 2, an introduction of grey system theory and model GM(1,1) are given. In Section 3, analysis of palladium price is proposed. The robustness and accuracy of the proposed method are investigated using experimental data. A conclusion is given in Section 4.

2 Grey system and GM(1,1) algorithm

Prediction is a process in which future values in the system are predicted based on current and past data. Generally, a predetermined mathematical model is used for accurate predictions. Today, various techniques are used for prediction. A more accurate the prediction is obtained when more information is available but, data collection requires time and cost. Spending more time, though, leads to an increase in the accuracy of predictions, but reduces the relevance and timeliness of information. On the other hand, all the required information is not always available, so it's important to use the techniques that can provide accurate information with the least information. The grey prediction method is a prediction method that can provide relatively accurate predictions with the least information. In this method, in order to predict a variable, we need to have information about the past intervals of the same variable and make predictions based on it.

Grey theory is a scientific field first introduced by Dang from the Huazhong University of Science and Technology [6]. This theory has gained popularity over the past two decades, because it only needs a limited amount of data to predict unknown systems. This theory has been used in various fields such as social sciences and economics, finance, technology, agriculture, industry, transportation, mechanics, meteorology, ecology, hydrology, geology, medicine, military, and so on.

Grey system theory consists of five main parts: grey prediction, grey relation, grey decision, grey scheduling and grey control. The core of Grey's theory is the grey prediction model. The main application of grey theory is low data and inadequate information. There are incomplete information in our social, economic, and scientific activities. This method predicts future values for time series measured at the same time intervals. The basis for predicting these models is based on the latest data set and all data have positive values and this data sequence is constant. The grey system characterizes the information in three different groups: black, white and grey. Black system is referred to unknown, white system is referred to well-known and grey system is referred to partly known information [7,12].

In grey systems theory, the GM(r,h) model is called as the grey prediction model in which r denotes the degree of differential equation and h denotes the number of variables in the model [10]. The classic grey forecast model is the GM(1,1) model, which is called the first-order grey model and used to predict the time series. The simplicity of modeling, implementation of the model, and the use of less time data to predict are the most important reasons for using the GM(1,1) model. The system is easily described by a first-order differential equation. We need a small amount of data (at least 4 observation points) to check and predict unreliable data and reduce errors. The accumulative generation operator (AGO) is used to smooth the randomness of the data. Solving the differential equation GM(1,1) predicts the values associated with n steps ahead of the system. Finally, using the predicted value and the inverse accumulative generation operator (I-AGO), the predicted value of the main data is computed [11]. The GM(1,1) algorithm is as follows:

1. Assume that $X^{(0)} = \{x^{(0)}(k)\}, \{k = 1, 2, \cdots, n : n \geq 4\}$ is a non-negative sequence of raw data and n is the size of the sample data, with the first order sequence of its accumulative generation operator equal to $X^{(1)} = \{x^{(1)}(k)\}$

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \qquad , k = 1, 2, \cdots, n \tag{1}$$

2. The average generated value of successive neighbors is obtained by the following formula

$$Z^{(1)} = \{z^{(1)}(k)\}, \qquad , k = 1, 2, \cdots, n \tag{2}$$

That

$$z^{(1)}(k) = \frac{1}{2}x^{(1)}(k-1) + \frac{1}{2}x^{(1)}(k) \tag{3}$$

3. The whitened equation of the GM(1.1) pattern will be as follows

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \tag{4}$$

By the discretization of equation 4, the differential equation is grey as follows

$$x^{(0)}(k) + az^{(1)}(k) = b \tag{5}$$

4. Calculate the values of a and b using the least squares estimation method

$$\hat{a} = \left(B^T B\right)^{-1} B^T Y \tag{6}$$

That

$$\hat{a} = \begin{bmatrix} a \\ b \end{bmatrix} \tag{7}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{8}$$

To prove equation 6, we replace all data values in equation 5

$$x^{(0)}(k) + az^{(1)}(k) = b$$

gives that

$$\begin{aligned}
&x^{(0)}(2)+az^{(1)}(2)=b,\\
&x^{(0)}(3)+az^{(1)}(3)=b,\\
&\cdots\cdots\cdots\\
&x^{(0)}(n)+az^{(1)}(n)=b.
\end{aligned}$$

That is,

$$Y=B\hat{a}.$$

For a pair of evaluated values a and b, the following error sequence is obtained using $-az^{(1)}(k)+b$ to replace $x^{(0)}(k), k=2,3,\cdots,n$,

$$\varepsilon=Y-B\hat{a}.$$

Let

$$\begin{aligned}
s=\varepsilon^T\varepsilon&=[Y-B\hat{a}]^T[Y-B\hat{a}]\\
&=\sum_{k=2}^{n}\left[x^{(0)}(k)+az^{(1)}(k)-b\right]^2.
\end{aligned}$$

The a and b values making s the minimum should satisfy

$$\begin{cases}\dfrac{\partial s}{\partial a}=2\sum_{k=2}^{n}[x^{(0)}(k)+az^{(1)}(k)-b]\cdot z^{(1)}(k)=0\\ \dfrac{\partial s}{\partial b}=-2\sum_{k=2}^{n}[x^{(0)}(k)+az^{(1)}(k)-b]=0.\end{cases}$$

That is,

$$\begin{cases}\sum_{k=2}^{n}[x^{(0)}(k)z^{(1)}(k)+a[z^{(1)}(k)]^2-b\cdot z^{(1)}(k)]=0\\ \sum_{k=2}^{n}[x^{(0)}(k)+az^{(1)}(k)-b]=0.\end{cases}$$

So, solving this system gives that

$$b=\frac{1}{n-1}\Big[\sum_{k=2}^{n}x^{(0)}(k)+a\sum_{k=2}^{n}z^{(1)}(k)\Big]$$

and

$$a=\frac{\frac{1}{n-1}\sum_{k=2}^{n}x^{(0)}(k)\cdot\sum_{k=2}^{n}z^{(1)}(k)-\sum_{k=2}^{n}x^{(0)}(k)\cdot z^{(1)}(k)}{\sum_{k=2}^{n}\left[z^{(1)}(k)\right]^2-\frac{1}{n-1}\Big[\sum_{k=2}^{n}z^{(1)}(k)\Big]^2}$$

From $Y=B\hat{a}$, it follows that

$$B^TB\hat{a}=B^TY,\qquad \hat{a}=[B^TB]^{-1}B^TY$$

But

$$B^TB = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}^T \cdot \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \sum_{k=2}^n \left[z^{(1)}(k)\right]^2 & -\sum_{k=2}^n z^{(1)}(k) \\ -\sum_{k=2}^n z^{(1)}(k) & n-1 \end{bmatrix},$$

$$\left[B^TB\right]^{-1} = \frac{\begin{bmatrix} n-1 & \sum_{k=2}^n z^{(1)}(k) \\ \sum_{k=2}^n z^{(1)}(k) & \sum_{k=2}^n \left[z^{(1)}(k)\right]^2 \end{bmatrix}}{n-1\sum_{k=2}^n \left[z^{(1)}(k)\right]^2 - \left[\sum_{k=2}^n z^{(1)}(k)\right]^2},$$

and

$$B^TY = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}^T \cdot \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -\sum_{k=2}^n x^{(0)}(k) \cdot z^{(1)}(k) \\ \sum_{k=2}^n x^{(0)}(k) \end{bmatrix}.$$

Therefore,

$$\begin{aligned}
\hat{a} &= \left[B^TB\right]^{-1} B^TY \\
&= \frac{\begin{bmatrix} -(n-1)\sum_{k=2}^n x^{(0)}(k) \cdot z^{(1)}(k) + \sum_{k=2}^n x^{(0)}(k) \cdot \sum_{k=2}^n z^{(1)}(k) \\ -\sum_{k=2}^n z^{(1)}(k) \cdot \sum_{k=2}^n x^{(0)}(k) z^{(1)}(k) + \sum_{k=2}^n x^{(0)}(k) \cdot \sum_{k=2}^n \left[z^{(1)}(k)\right]^2 \end{bmatrix}}{(n-1)\sum_{k=2}^n \left[z^{(1)}(k)\right]^2 - \left[\sum_{k=2}^n z^{(1)}(k)\right]^2} \\
&= \begin{bmatrix} \dfrac{\frac{1}{n-1}\sum_{k=2}^n x^{(0)}(k) \cdot \sum_{k=2}^n z^{(1)}(k) - \sum_{k=2}^n x^{(0)}(k) \cdot z^{(1)}(k)}{\sum_{k=2}^n \left[z^{(1)}(k)\right]^2 - \frac{1}{n-1}\left[\sum_{k=2}^n z^{(1)}(k)\right]^2} \\ \frac{1}{n-1}\left[\sum_{k=2}^n x^{(0)}(k) + a\sum_{k=2}^n z^{(1)}(k)\right] \end{bmatrix} \\
&= \begin{bmatrix} a & b \end{bmatrix}^T.
\end{aligned}$$

5. Calculate the time response of the GM(1,1) equation based on the values a and b

$$\hat{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{b}{a}]e^{-ak} + \frac{b}{a} \tag{9}$$

6. The reconstructed values of raw data are obtained by the following equation

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \tag{10}$$

Table 1: $MAPE = 1.2\%$, Palladium price prediction

months	real data	predicted data
2016 January	470.45	470.45
2016 March	566.85	554.7778
2016 May	584.1	587.0291
2016 July	627.4	621.1553
2016 September	657.95	657.2654
2016 November	699.8	695.4747
2017 January	739	735.9052
2017 March	771.15	778.6861
2017 May	797.95	823.9540
2017 July	854.75	871.8535
2017 September	939.39	922.5376
2017 November	992.5	976.1682

The parameter a is the development index and b is the grey actuating quantity. The prediction accuracy of the GM(1,1) model depends on the values a and b and the selection of the initial conditions during the modeling process GM(1,1). In order to improve the accuracy of this method, selecting the initial values of the parameter is important [13].

3 Data and numerical results

In this study, to obtain information and data on the price of palladium, the website *www.tgju.org/en* is used. The method GM(1,1) is applied on data to predict palladium price. the mean average percentage error ($MAPE$) was used to determine the accuracy of prediction as follows

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\frac{\mid y_i - \hat{y_i} \mid}{y_i}, \tag{11}$$

where N is number of data, y_i is the i^{th} data and $\hat{y_i}$ is its prediction.

In this study, the average of palladium price is considered for each two months between the 2016 January to 2017 November. The four initial data are considered as input and fifth data is predicted. This trend is continued for 2 years. The results show that the error is $MAPE = 1.2\%$ which is acceptable.

4 Conclusion

This papers investigated forecasting of palladium price using grey system theory. The palladium price between the period 2016 January and 2017 November are considered. It is shown that the method is accurate and $MAPE$ is 1.2% for two years predictions. For the future works, one can consider the price of other metals of PGM groups and analysis the relationship between palladium price and the price of other elements of this group. In addition, one can investigate the effect of other parameters like as the demand of palladium in different industries to the its price.

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